

EE8552 POWER ELECTRONICS L T P C 3 0 0 3

OBJECTIVES:

1. To impart knowledge on the following Topics different types of power semiconductor devices and their switching.
2. Operation, characteristics and performance parameters of controlled rectifiers.
3. Operation, switching techniques and basics topologies of DC-DC switching regulators.
4. Different modulation techniques of pulse width modulated inverters and to understand harmonic reduction methods.
5. Operation of AC voltage controller and various configurations.

UNIT I POWER SEMI-CONDUCTOR DEVICES 9

Study of switching devices, SCR, TRIAC, GTO, BJT, MOSFET, IGBT and IGCT- Static characteristics: SCR, MOSFET and IGBT - Triggering and commutation circuit for SCR Introduction to Driver and snubber circuits.

UNIT II PHASE-CONTROLLED CONVERTERS 9

2-pulse, 3-pulse and 6-pulse converters— performance parameters —Effect of source inductance— Firing Schemes for converter—Dual converters, Applications—light dimmer, Excitation system, Solar PV systems.

UNIT III DC TO DC CONVERTERS 9

Step-down and step-up chopper-control strategy— Introduction to types of choppers-A, B, C, D and E -Switched mode regulators- Buck, Boost, Buck- Boost regulator, Introduction to Resonant Converters, Applications-Battery operated vehicles.

UNIT IV INVERTERS 9

Single phase and three phase voltage source inverters (both 1200 mode and 1800 mode)— Voltage & harmonic control—PWM techniques: Multiple PWM, Sinusoidal PWM, modified sinusoidal PWM — Introduction to space vector modulation —Current source inverter, Applications-Induction heating, UPS.

UNIT V AC TO AC CONVERTERS 9

Single phase and Three phase AC voltage controllers—Control strategy- Power Factor Control — Multistage sequence control -single phase and three phase cyclo converters — Introduction to Matrix converters, Applications —welding .

TOTAL : 45 PERIODS

UNIT - I

Study of switching devices, Diode, SCR, TRIAC, GTO, BJT, MOSFET, IGBT, ^{IGCT} - static and dynamic characteristics - Triggering and commutation circuit for SCR - Design of Driver and snubber circuit.

Introduction :

Power electronics is a subject that concerns the application of electronic principles into situations that are rated at power level rather than signal level.

Subject that deals with the apparatus and equipment working on the principle of electronics but rated at power level rather than signal level.

Applications of Power Electronics :Aerospace :

space shuttle power supplies, satellite power supplies, aircraft power systems.

Commercial :

Advertising, heating, air conditioning, central refrigeration, computer and office equipment, UPS, elevators.

Industrial :

arc and industrial furnaces, blowers and fans, pumps and compressors, rolling mills, textile mills, cement mills.

Residential :

Air conditioning, cooking, lighting, space heating, refrigerators, dryers, fans, personal computers.

vacuum cleaners, washing and sewing machines, food mixers.

1.3 Telecommunication: central, mobile, cordless, etc.

Battery chargers, Power Supplies (DC and UPS)

1.4 Transportation: Battery chargers, traction control of electric

vehicles, trolley buses, subways.

Utility systems:

HVDC, static circuit Breakers, fans and boiler feed pumps.

ADVANTAGES:

- (i) High efficiency due to low loss in power semiconductor device.
- (ii) High reliability
- (iii) Long life and less maintenance
- (iv) Fast dynamic response
- v) small size and less weight and lower installation cost.
- vi) Mass production of power semiconductor devices

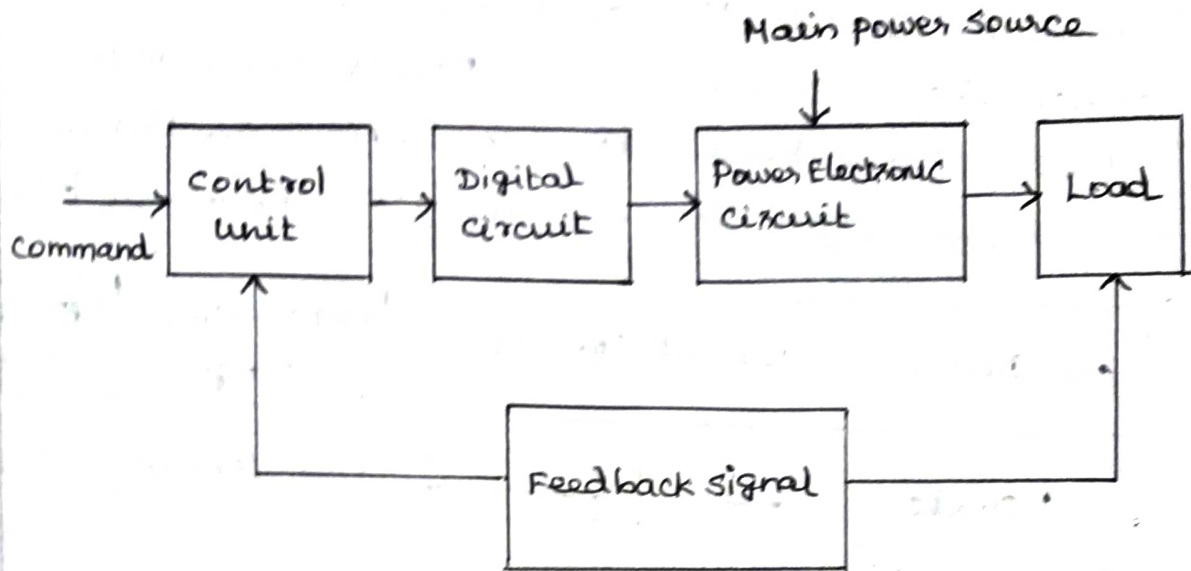
Disadvantages:-

- (i) Power electronic converter circuits have a tendency to generate harmonics in the supply system.
- (ii) AC to DC and AC to AC converters operate at a low input power factor under certain operating conditions, in order to avoid a low pf some special measures have to be adopted.
- iii) Power electronic controllers have low overload capacity.

(iv) Regeneration of power is difficult in power electronic converter systems.

Power Electronic Systems :-
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The major components of a power electronic system are shown in figure.



The output from the power electronic circuit may be variable dc or ac voltage, or it may be a variable voltage and frequency. The output of a power electronic converter circuit depends upon the requirements of the load.







For example, if the load is a dc motor, the converter output must be adjustable direct voltage. In case load is a 3 phase induction motor, the converter may have adjustable voltage and frequency at its output terminals.

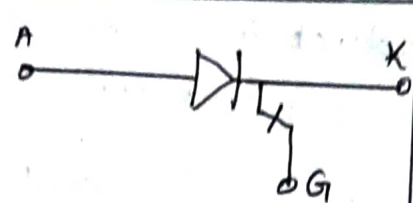
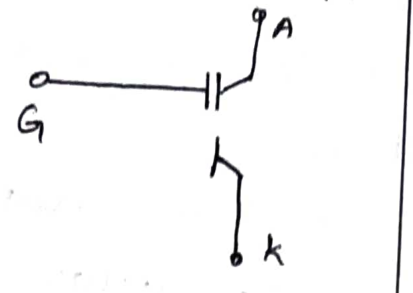
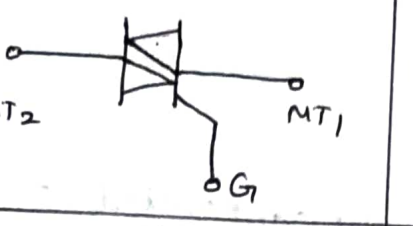
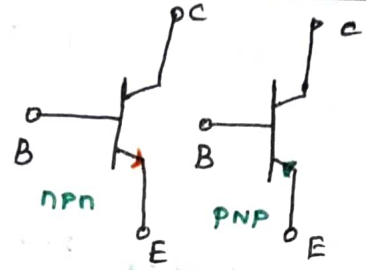
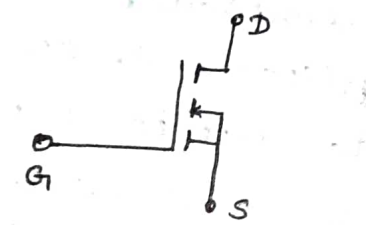
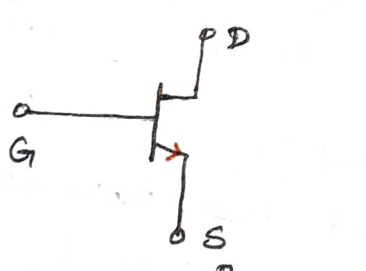
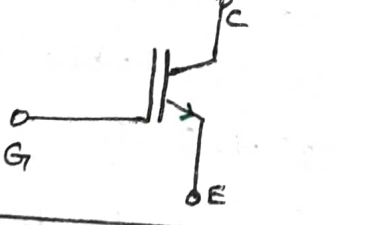
The feedback component in figure measures a parameter of the load, e.g. speed in case of a rotating machine, and compares it with the command. The difference of the two, through the

digital circuit components, controls the instant of turn-on of semiconductor devices forming the solid state power converter system.

Power semiconductor Devices :-
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Silicon controlled Rectifier (SCR) was first introduced in 1957 as a power semiconductor device. since then, several other power semiconductor devices have been developed. Most of these semiconductor devices are listed below in table.

S.NO	Device	circuit symbol	Voltage/Current ratings	Upper operating freq (KHz)
1.	Diode		5000V/5000A	1.0
2.	THYRISTOR			
	(a) SCR		7000V/5000A	1.0
	(b) LASCR		6000V/3000A	1.0
	(c) ASCR/ RCT		2500V/400A	2.0
	(d) GTO		5000V/3000A	2.0
	(con)			

S.No	Device	circuit symbol	voltage/ current ratings	upper operating freq (K.Hz)
	(e) SITH		2500V/ 500A	100.0
	(f) MCT		1200V/40 A	20.0
	(g) Triac		1200V/ 1000A	0.50
3.	Transistors			
	(a) BJT		1400V/ 400A	10.0
	(b) MOSFET (n-channel)		1000V/ 50A	100.0
	(c) SIT		1200V/ 300A	100.0
	(d) IGBT		1200V/ 500A	50.0

SCR - Silicon Controlled Rectifier

LASCR - Light Activated SCR

ASCR - Asymmetrical SCR

RCT - Reverse Conducting Thyristor

GTO - Gate Turn Off Thyristor.

SITH - Static Induction Thyristor.

MCT - Mos Controlled Thyristor.

BJT - Bipolar Junction Transistor.

MOSFET - Metal Oxide Semiconductor Field Effect Transistor.

SIT - Static Induction Transistor.

IGBT - Insulated Gate Bipolar Transistor.

CLASSIFICATION OF POWER SEMICONDUCTOR DEVICES

Diodes :

These are uncontrolled rectifying devices. Their ON and OFF states are controlled by power supply.

Thyristors :

These have controlled turn on by a gate signal. After thyristors are ON, they remain latched in on-state due to internal regenerative action and gate loses control. These can be turned off by the power circuit.

Controllable switches :-

These devices are turned on and turned off by the application of control signals. The devices which behaves as controllable switches are BJT, MOSFET, GTO, SITH, IGBT, SIT and MCT.

TRIAC and RCT possess bi-directional current capability, all other remaining devices are uni-directional current devices.

Study of Switching Devices - Diode :

Power Diode :-

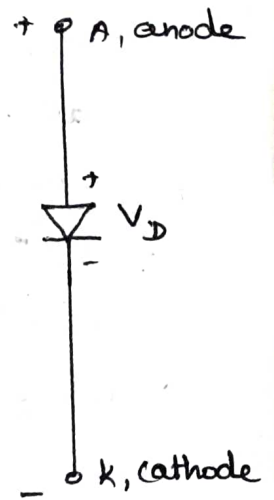
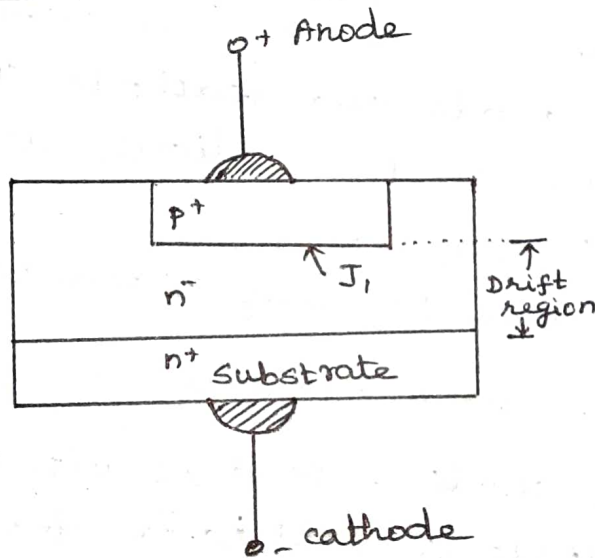
A low power diode called signal diode, is a pn-junction device. A high Power diode, called Power diode is also a pn-junction device but with constructional features somewhat different from a signal diode.

The voltage, current and power ratings of Power diodes and transistors are much higher than the corresponding ratings for signal devices. Power devices operate at lower switching speeds whereas signal diodes and transistors operate at higher switching speeds.

BASIC STRUCTURE OF POWER DIODES :

carrying density

P	N	Junction
10^{16} to 10^{17} cm^{-3}	same	P^+N
10^{19} cm^{-3}	10^{17} cm^{-3}	P^+N
10^{19} cm^{-3}	10^{13}	P^+N^-
10^{19} cm^{-3}	10^{19} cm^{-3}	P^+N^+



(a) Structural features of Power diode

circuit symbol.

It consists of heavily doped n^+ substrate. On this substrate, a lightly doped n^- layer is epitaxially grown. Now a heavily doped p^+ layer is diffused into n^- layer to form the anode of power diode.

n^- layer is the basic structural feature not found in signal diodes.

The function of n^- layer is to absorb the depletion layer of the reverse biased p^n junction J_1 . The break down voltage needed in a power diode governs the thickness of n^- layer. Greater the breakdown voltage, more the n^- layer thickness.

The drawback of n^- layer is to add significant ohmic resistance to the diode when it is conducting a forward current. This leads to large power dissipation in the diode.

CHARACTERISTICS OF POWER DIODES :

Power diode is a two-terminal, $p-n$ semiconductor device. The two terminals of diode are called anode and cathode. Two important characteristics of power diodes are

- (1) Diode $V-I$ characteristics.
- (2) Diode Reverse Recovery characteristics.
- (3) Diode $V-B$ characteristics

When anode is positive with respect to cathode, diode is said to be forward biased. With increase of the source voltage V_s from zero value, initially diode current is zero. From $V_s = 0$ to cut in voltage, the forward diode current is very small. Cut in voltage is also known as threshold voltage or turn on voltage. Beyond cut in voltage, the diode current rises rapidly and the diode is said to conduct.

T = absolute temperature in kelvin ($K = 273 + C^\circ$)

k = Boltzmann's constant; $1.3806 \times 10^{-23} \text{ J/K}$.

When cathode is positive with respect to anode, the diode is said to be reverse biased. In the reverse biased condition, a small reverse current called leakage current of the order of microamperes or milliamperes flows.

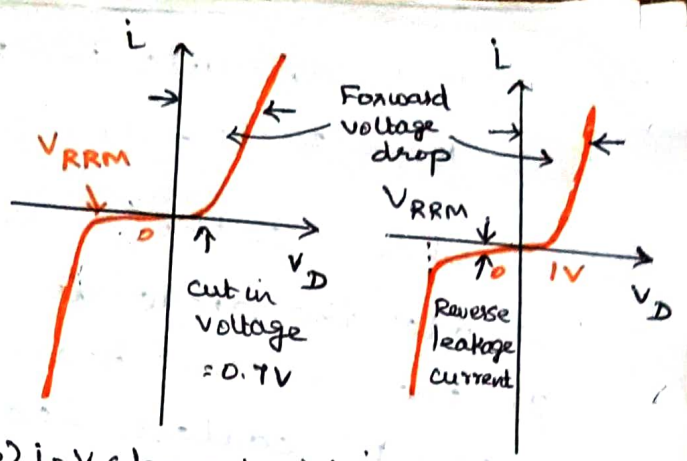
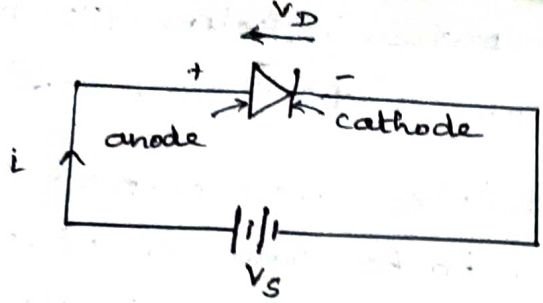
At reverse breakdown, voltage remains const, but reverse current becomes quite high limited only by the external circuit resistance.

A large reverse breakdown voltage, associated with high reverse current leads to excessive power loss that may destroy the diode.

For an ideal diode, $V_D = 0$, reverse leakage current = 0, cut in voltage = 0, reverse breakdown voltage V_{RRM} is infinite.

Peak inverse voltage (PIV) is the largest reverse voltage to which a diode may be subjected during its working. PIV is the same as V_{RRM} .

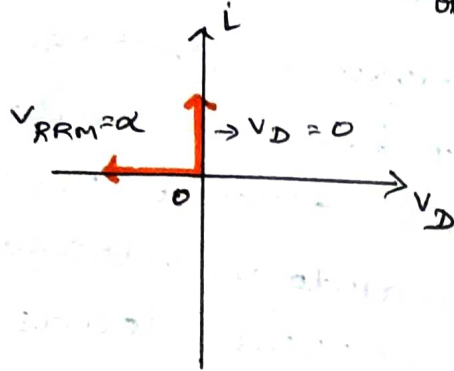
The power diodes are now available with forward current ratings of 1A to several thousands amperes and with reverse voltage ratings of 50V to 5000V or more.



(a) Forward biased Power diode

b) i-v characteristics of signal diode

c) i-v characteristics of power diode.



d) i-v characteristics of ideal diode

For silicon diode, the cut in voltage is around 0.7V. When diode conducts, there is a forward voltage drop of the order of 0.8 to 1V.

The characteristics can be expressed by an equation known as Shockley diode equation, and is given by

$$I_D = I_S [e^{V_D/nV_T} - 1]$$

where I_D = current through the diode A
 V_D = Diode voltage with anode positive with respect to cathode (V).

I_S = leakage current range: 10^{-6} to 10^{-15} A.

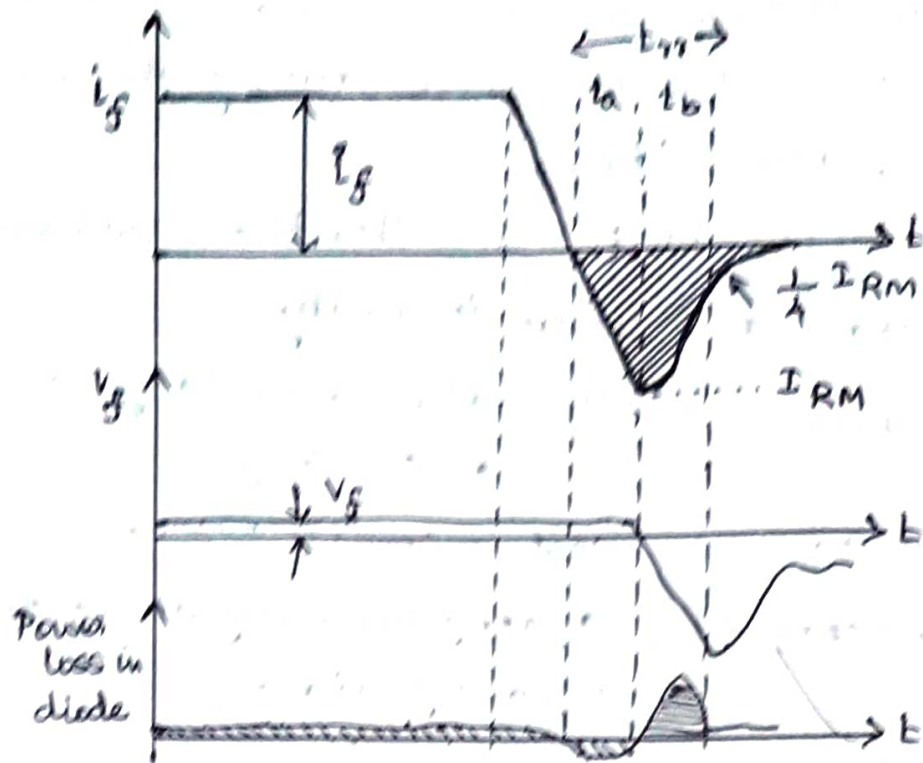
n = emission co-efficient, value: 1 to 2.

V_T = Thermal voltage, it is given by

$$V_T = \frac{KT}{q}$$

where q = electron charge 1.6022×10^{-19} C.

(10) DIODE REVERSE RECOVERY CHARACTERISTICS :-



⇒ After the forward diode current decays to zero, the diode continues to conduct in the reverse direction because of the presence of stored charges in the depletion region.

⇒ Reverse recovery time t_{rr} is defined as the time between the instant forward diode current becomes zero, and the instant reverse recovery current decays to 25% of its reverse peak value I_{RM} .

$$\Rightarrow t_{rr} = t_a + t_b$$

Time t_a = Time between zero crossing of forward current and peak reverse current I_{RM} .
∴ charge stored in depletion layer is removed.

Time t_b = Measured from the instant of reverse peak value I_{RM} to the instant when $0.25 I_{RM}$ is reached. Charge from the semiconductor layer is removed.

⇒ The shaded area in fig (a) represents the stored charge or reverse recovery charge Q_R which must be removed during the reverse recovery time (t_{rr}).

⇒ The ratio t_b/t_a is called the softness factor (or) S-factor. Its value is unity.

S factor small - diode has large oscillatory over voltage

S factor = 1, soft recovery diode

S factor < 1, snappy recovery diode or fast recovery diode.

Peak reverse current I_{RM} can be expressed as

$$I_{RM} = t_a \cdot \frac{di}{dt} \quad \dots \dots \dots (1)$$

$\frac{di}{dt}$ = rate of change of reverse current.

$$Q_R = \frac{1}{2} I_{RM} \times t_{rr}$$

$$I_{RM} = \frac{2 Q_R}{t_{rr}} \quad \dots \dots \dots (2)$$

If $t_{rr} \equiv t_a$, from eqn ①

$$I_{RM} = t_{rr} \cdot \frac{di}{dt} \quad \dots \dots \dots (3)$$

From eqn ② & ③, we get,

$$t_{rr} \cdot \frac{di}{dt} = \frac{2 Q_R}{t_{rr}}$$

$$t_{rr} = \left[\frac{2 Q_R}{di/dt} \right]^{1/2}$$

From eqn ③, $I_{RM} = \left[\frac{2 Q_R}{di/dt} \right]^{1/2} \cdot di/dt$

$$I_{RM} = \left[2 Q_R \frac{di}{dt} \right]^{1/2}$$

The time t_s required to remove these excess carriers is called storage time and only after base current I_B begins to decrease towards zero. During t_s , collector current falls from I_{CS} to $0.9I_{CS}$ collector-emitter voltage V_{CE} rises from V_{CES} to $0.1V_{CE}$.

\Rightarrow After t_s , collector current begins to fall and collector-emitter voltage starts building up.

t_f - fall time at which I_C drops from $0.9I_{CS}$ to $0.1I_{CS}$, V_{CE} rises from $0.1V_{CE}$ to $0.9V_{CE}$.

$$t_{off} = t_s + t_f$$

t_n = conduction period of transistor,

t_o = off period.

$T = 1/f$ periodic time,

f = switching frequency.

STUDY OF SWITCHING DEVICES - SCR

Constructional Details:

Thyristor is a four layer, three-junction, P-N-P-N semiconductor switching device. It has 3 terminals: anode, cathode and gate. Basically a thyristor consists of four layers of alternate P-type and N-type silicon semiconductors forming three junctions J_1 , J_2 and J_3 . Gate terminal is usually kept near the cathode terminal.

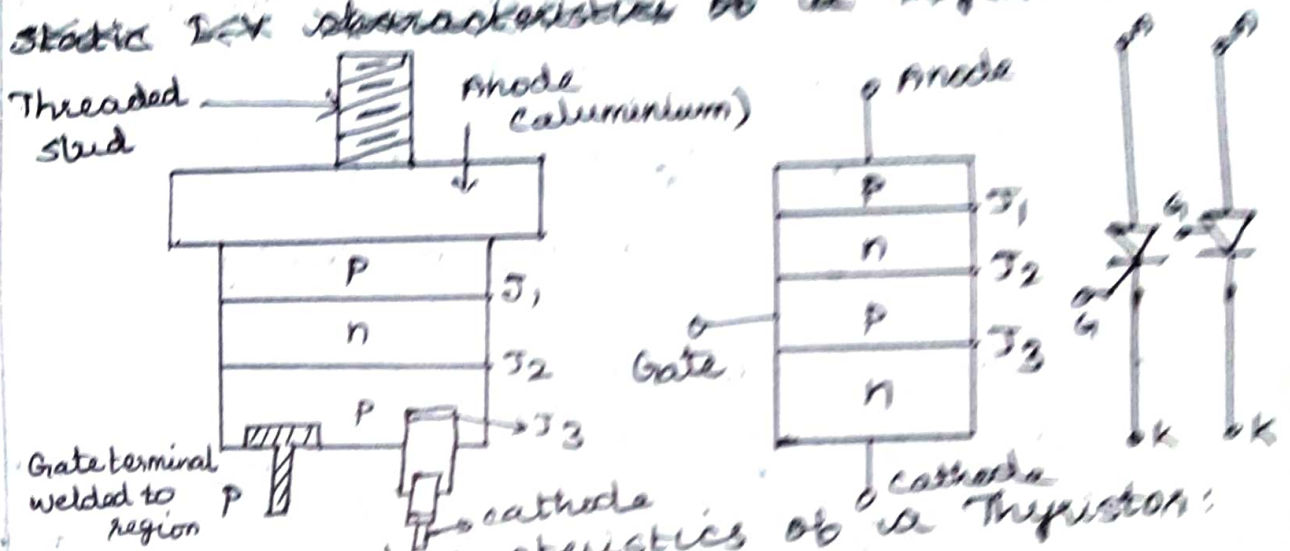
The terminal connected to outer P region is called Anode (A), the terminal connected to outer N region is called Cathode and connected to inner P-region is called the Gate (G).

For large current applications thyristors need better cooling, this is achieved by mounting them onto heat sinks.

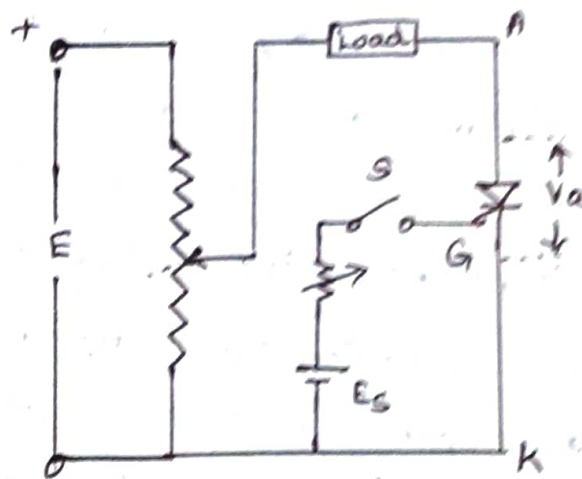
SCRs of voltage rating 10kV and an RMS current rating of 3000 A, power handling capacity 30W are available. As SCRs are solid state devices, they are compact, possess high reliability and have low loss.

An SCR is an unidirectional device that blocks the current flow from cathode to anode.

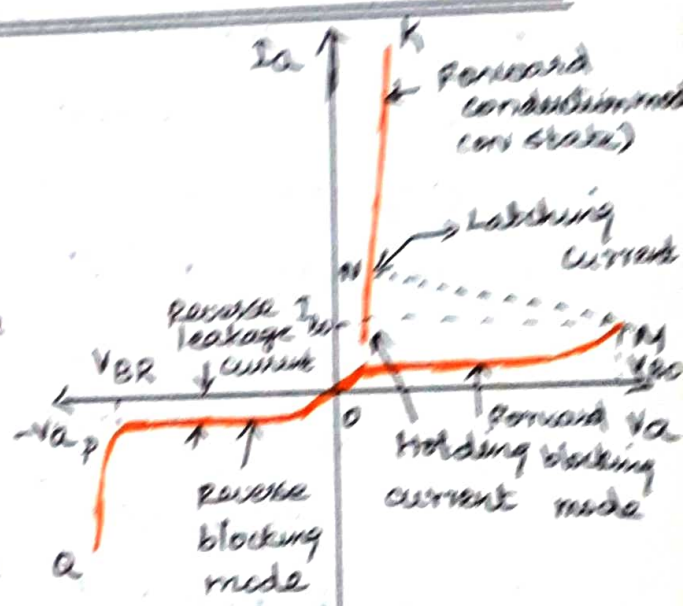
Static I-V characteristics of a Thyristor:



Static I-V characteristics of a Thyristor:



Circuit for obtaining thyristor I-V characteristics



V_{BO} - Forward Breakover voltage
 V_{BR} - Reverse Breakdown voltage

Thyristor has three basic modes of operation

- i) Reverse blocking mode
- ii) Forward blocking mode (off state)
- iii) Forward conduction mode (on state).

(i) Reverse Blocking mode :-

[When cathode is made positive with respect to anode, with switch 'S' open, thyristor is reverse biased. Junctions J_1, J_3 are to be reverse biased whereas junction J_2 is forward biased. A small leakage current of the order of a few milliamperes flows. This is reverse blocking mode called the off-state of the thyristor. This is shown by OP.

If the reverse voltage is increased, at a critical breakdown level, called reverse breakdown voltage V_{BR} , an avalanche occurs at J_1 and J_3 and the reverse current increases rapidly. Maximum working reverse voltage across a thyristor does not exceed V_{BR} . Reverse avalanche region is shown by PQ. The SCR in the reverse blocking mode may therefore be treated as an open switch.

Forward Blocking Mode :

When anode is positive, with respect to the cathode, with gate circuit open thyristor is said to be forward biased. Junctions J_1, J_3 are forward biased, but junction J_2 is reverse biased

OM represents the forward blocking mode. The forward leakage current is small, SCR offers a high impedance.

Forward Conduction mode:

When anode to cathode forward voltage is increased with gate circuit open, reverse biased junction J_2 will have an avalanche break-down at a voltage called forward breakover voltage V_{BO} . After this breakdown, thyristor gets turned on with point M at once shifting to N. Here NH represents the forward conduction mode.

A thyristor can be brought from forward blocking mode to forward conduction mode by turning it on by applying (i) a +ve gate pulse between gate and cathode.
ii) a forward breakover voltage across anode and cathode. In forward conduction mode thyristor is treated as a closed switch.

SWITCHING CHARACTERISTICS OF THYRISTORS: Switching characteristics during Turn-on:

Thyristor turn-on time, is defined as the time during which it changes from forward blocking state to final on-state. Total turn-on time can be divided into 3 intervals

- (i) Delay time (t_d)
- (ii) Rise time (t_r)
- (iii) Spread time (t_p).

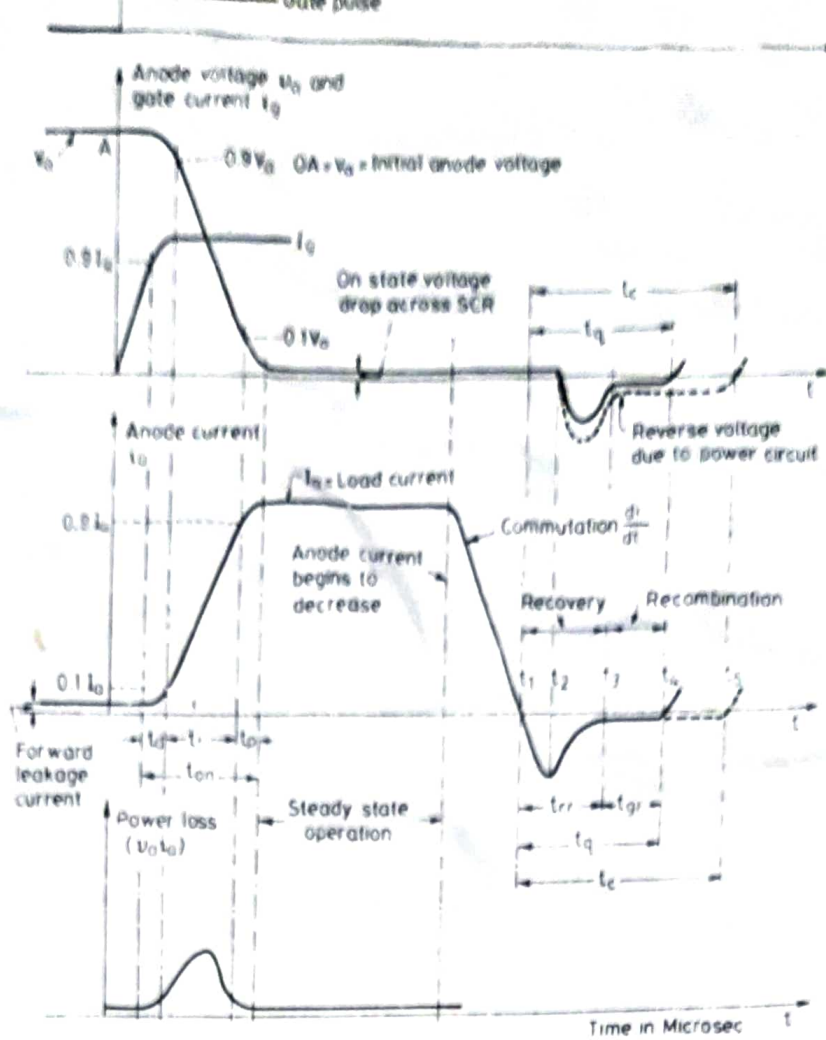


Fig. 4.8. Thyristor voltage and current waveforms during turn-on and turn-off processes

* This can be achieved through natural commutation or forced commutation

Delay time (t_d) :-

The delay time (t_d) is measured from the instant at which gate current reaches $0.9I_g$ to the instant at which anode current reaches $0.1I_a$. I_a , I_g are final values of anode current and gate current. The thyristor initially in the forward blocking state, the anode voltage is OA and anode current is small leakage current. Initiation of turn on process is indicated by a rise in anode current from small forward leakage current and a fall in anode-cathode voltage from forward blocking voltage OA . The delay time can be decreased by applying high gate current and more forward voltage between anode & Cathode.

Rise time (t_r) :-

The rise time t_r is the time taken by the anode current to rise from $0.1 I_a$ to $0.9 I_a$. The rise time is inversely proportional to the magnitude of gate current and its build up rate. t_r can be reduced if high and steep current pulses are applied to the gate. As the rise time is small, the anode current is not able to spread over the entire cross section of cathode. During rise time, turn-on losses in the thyristor are the highest due to high anode voltage (V_a) and large anode current (I_a) occurring together in the thyristor.

Spread time (t_p) :-

The spread time is the time taken by the anode current to rise from $0.9 I_a$ to I_a . During this time, conduction spreads over the entire cross section of the cathode of SCR. After the spread time, anode current attains steady state value and the voltage drop across ~~CRS~~^{SCR} is equal to the on-state voltage drop of the order of 1 to 1.5V.

Total turn on time is equal to the sum of delay time, rise time and spread time. Total turn-on time depends upon the anode circuit parameters and the gate signal waveshapes. Turn on time can be reduced by using higher values of gate currents.

Switching characteristics during Turn-off :-

Thyristor turn-off means that it has changed from ON to OFF state and is capable of blocking the forward voltage. Once the thyristor is ON, gate loses control. The SCR can be turned off by reducing the anode current below holding current.

The turn-off time (t_q) of a thyristor is defined as the time between the instant anode current becomes zero and the instant SCR regains forward blocking capability. During time t_q , all the excess carriers from the four layers of SCR must be removed. This removal of excess carriers consists of sweeping out of holes from outer P-layer and electrons from outer n-layer. The carriers around junction J_2 can be removed only by recombination. The turn-off time is divided into two intervals; reverse recovery time (t_{rr}), Gate recovery time (t_{gr})

$$t_q = t_{rr} + t_{gr}$$

At instant t_1 , anode current becomes zero. After t_1 , anode current builds up in the reverse direction with the same di/dt slope as before t_1 . The reason for the reversal of anode current after t_1 is due to the presence of carriers stored in four layers. The reverse recovery current removes excess carriers from the end junctions J_1 & J_3 between instants t_1 & t_2 .

At instant t_2 , when about 60% of the stored charges are removed from the outer two layers, carrier density across J_1 and J_3 begins to decrease and with this reverse recovery current also starts decaying.

The fast decay of recovery current causes a reverse voltage across the device due to the circuit inductance. This voltage surge appears across the thyristor terminals and may therefore damage it. In practice, this is avoided by using protective RC elements across SCR.

At instant t_3 , when reverse recovery current has fallen to nearly zero value, end junctions J_1 and J_3 recover and SCR is able to block the reverse voltage.

At the end of reverse recovery period ($t_3 - t_1$), the middle junction J_2 still has trapped charges, therefore, the thyristor is not able to block the forward voltage at t_3 . The trapped charges around J_2 , in the inner two layers, cannot flow to the external circuit. These trapped charges must decay only by recombination. The time for the recombination of charges between t_3 and t_4 is called gate recovery time t_{gr} .

At instant t_4 , junction J_2 recovers and the forward voltage can be reapplied between anode and cathode. t_{gr} is in the range of 3 to 100 μsec .

Power Transistors are turned on when a current signal is given to base, when this control signal is removed, a power transistor is turned off.

Power Transistors are of four types

- (i) Bipolar Junction Transistors (BJTs)
- (ii) Metal oxide semiconductor field effect Transistors (MOSFETs)
- (iii) Insulated Gate Bipolar Transistors (IGBT)
- (iv) Static Induction Transistors (SITs).

Bipolar Junction Transistors (BJT) :-

A bipolar transistor is a three-layer, two junction npn or pnp semiconductor device.

One p-region sandwiched by two n-regions, npn transistor is obtained.

One n-region sandwiched by two p-regions, pnp transistor is obtained.

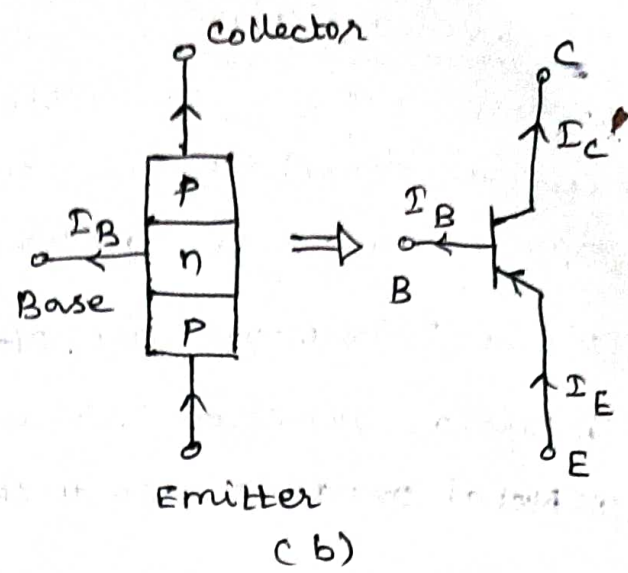
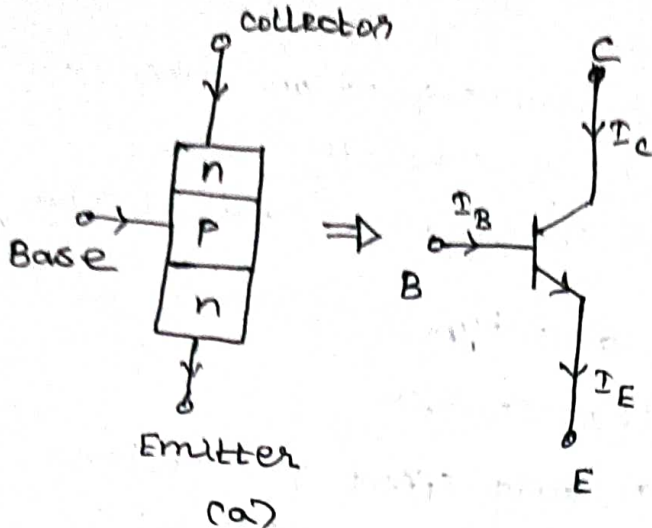
The term "bipolar" denotes that the current flow in the device is due to the movement of both holes and electrons.

A BJT has 3 terminals named

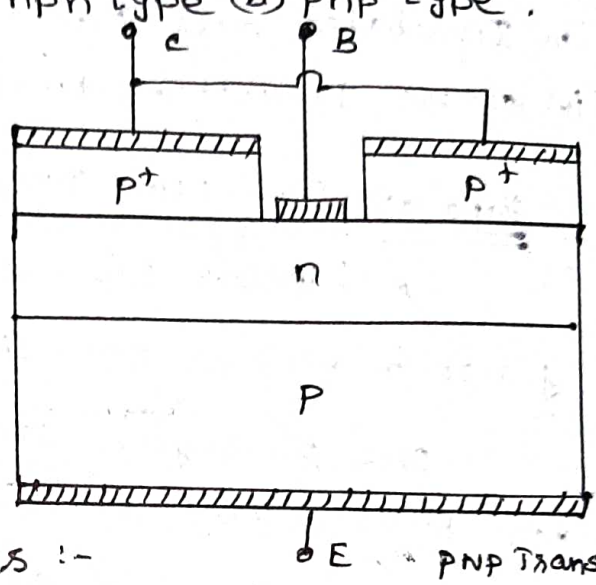
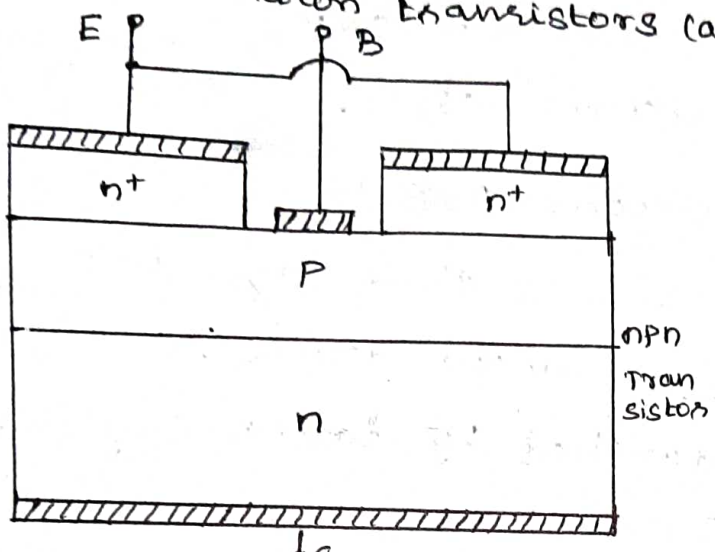
- i) collector (C)
- ii) Emitter (E)
- iii) base (B)

⇒ An emitter is indicated by an arrowhead indicating the direction of emitter current.

⇒ NPN transistors is very wide in high voltage and high current applications.



Bipolar Junction Transistors (a) npn type (b) pnp type.



steady state characteristics :-

There are 3 possible configurations

- i) Common collector
- ii) Common Base
- iii) Common Emitter

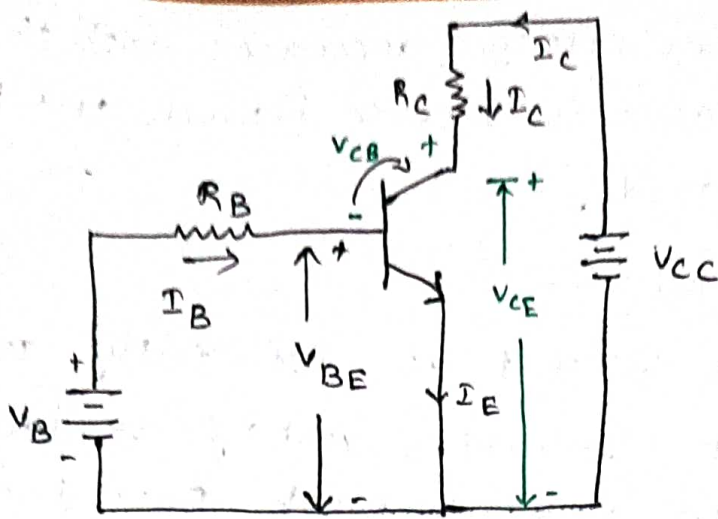
Common emitter arrangement is more common in switching applications.

Input characteristics :

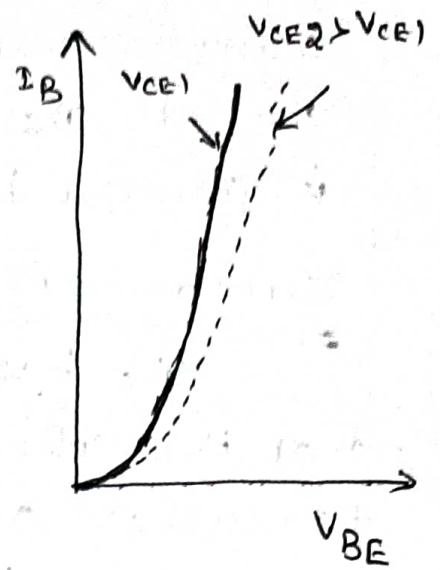
Base current I_B against base emitter voltage V_{BE} .

Output characteristics :

Collector current I_C against collector-emitter voltage V_{CE}

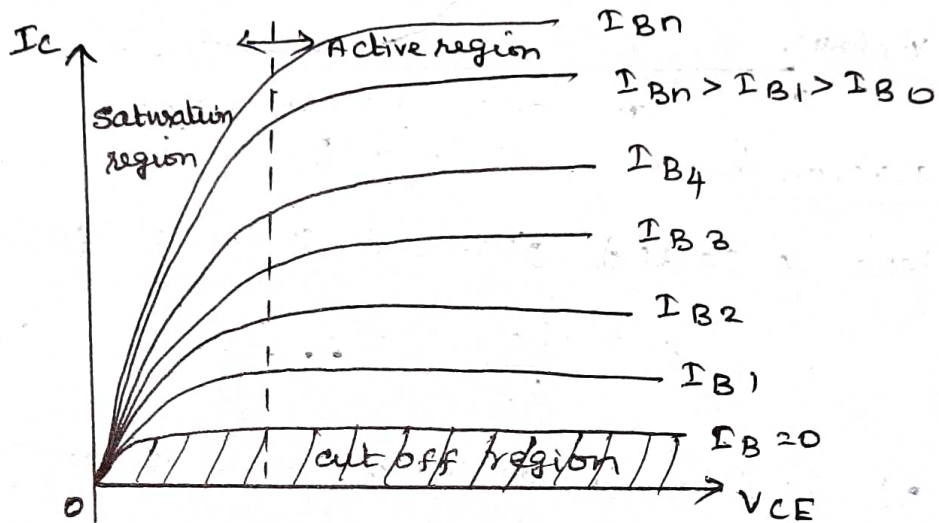


npn transistor circuit



Input Characteristics.

I_B versus V_{BE} graph resembles a diode curve, when collector-emitter voltage V_{CE2} is more than V_{CE1} , base current for the same V_{BE} decreases.



output characteristics

There are 3 operating regions of a Transistor: cut off, active and saturation.

cut off Region:

The transistor is off or the base current is not enough to turn it on and both junctions are reverse biased.

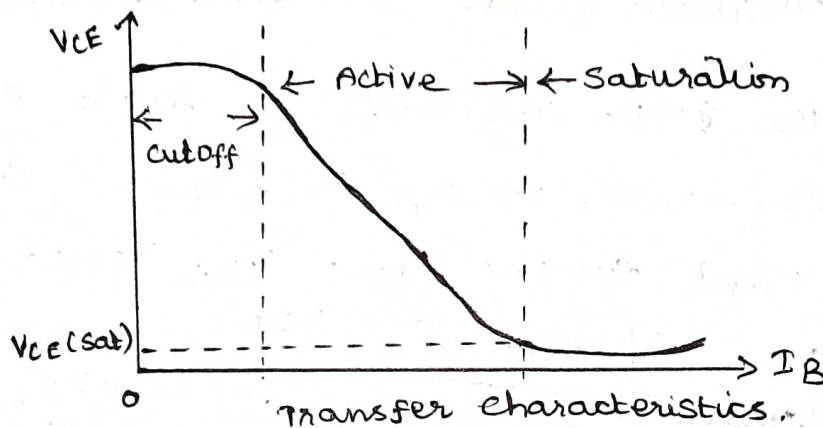
Active Region:

The transistor acts as an amplifier, where the base current is amplified by a gain.

collector-emitter voltage decreases with the base current. The CBJ is reverse biased, and the BEJ is forward biased.

Saturation Region :

The base current is sufficiently large so that the collector-emitter voltage is low. The transistor acts as a switch. Both junctions are forward biased.



Relation between α and β :

Here α called forward current gain, is defined as

$$\alpha = \frac{I_C}{I_E} \quad \dots \dots \dots (1)$$

As $I_C < I_E$, value of α varies from 0.95 to 0.99.

In a transistor, base current is effectively the input current, collector current is the output current β is called current gain.

$$\beta = \frac{I_C}{I_B} \quad \dots \dots \dots (2)$$

I_B is much smaller, $\beta > 1$ and varies from 50 to 300. In another system of analysis called h parameters, h_{FE} is used in place of β .

$$\beta = h_{FE} = \frac{I_C}{I_B} \quad \dots \dots \dots (3)$$

use of KCL from npn transistor circuit figure,

$$I_E = I_C + I_B \dots \dots \dots (4)$$

Emitter current is the largest of the 3 currents, collector current is almost equal but less than emitter current, Base current has the least value.

Dividing both sides of eqn (4) by I_C we get,

$$\frac{I_E}{I_C} = \frac{I_C}{I_C} + \frac{I_B}{I_C}$$

$$\frac{1}{\alpha} = 1 + \frac{1}{\beta}$$

$$\frac{1}{\alpha} = \frac{\beta + 1}{\beta}$$

$\beta \neq$

$$\alpha = \frac{\beta}{1 + \beta} \dots \dots \dots (5)$$

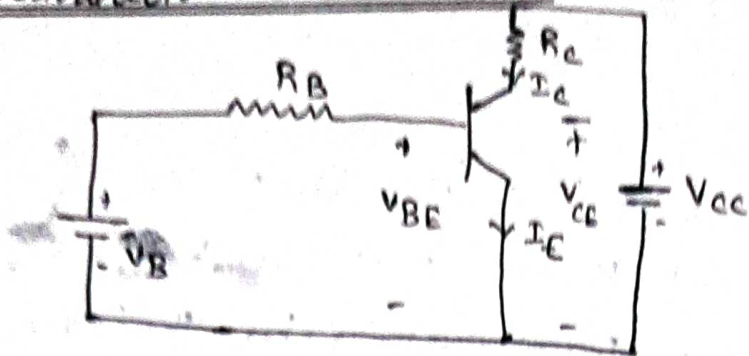
$$\frac{1}{\alpha} = 1 + \frac{1}{\beta}$$

$$\frac{1}{\beta} = \frac{1}{\alpha} - 1$$

$$\frac{1}{\beta} = \frac{1 - \alpha}{\alpha}$$

$$\beta = \frac{\alpha}{1 - \alpha} \dots \dots \dots (6)$$

Transistor As a switch:



$$V_B = I_B R_B + V_{BE}$$

$$\frac{V_B - V_{BE}}{R_B} = I_B$$

From the above figure,

$$V_B = I_B R_B + V_{BE} \quad \dots \dots \dots (7)$$

$$I_B = \frac{V_B - V_{BE}}{R_B}$$

$$V_{CC} = I_C R_C + V_{CE}$$

$$V_C = V_{CE} = V_{CC} - I_C R_C \quad \beta = \frac{I_C}{I_B}$$

$$= V_{CC} - \beta I_B R_C$$

$$= V_{CC} - \frac{\beta R_C}{R_B} (V_B - V_{BE})$$

$$\left[\begin{array}{l} V_{CE} = V_{CB} + V_{BE} \\ V_{CB} = V_{CE} - V_{BE} \end{array} \right] \quad (\otimes)$$

If V_{CES} is the collector-emitter saturation voltage, then collector current I_{CS} is given by,

$$V_{CC} = I_{CS} R_C + V_{CES}$$

$$I_{CS} = \frac{V_{CC} - V_{CES}}{R_C}$$

Corresponding value of base current,

$$I_{BS} = \frac{I_{CS}}{\beta}$$

The circuit is designed so that I_B is higher than I_{BS} . The ratio of I_B to I_{BS} is called the over drive factor (ODF).

$$O.D.F = \frac{I_B}{I_{B_s}}$$

The ratio of I_{C_s} to I_B is called forced β ,

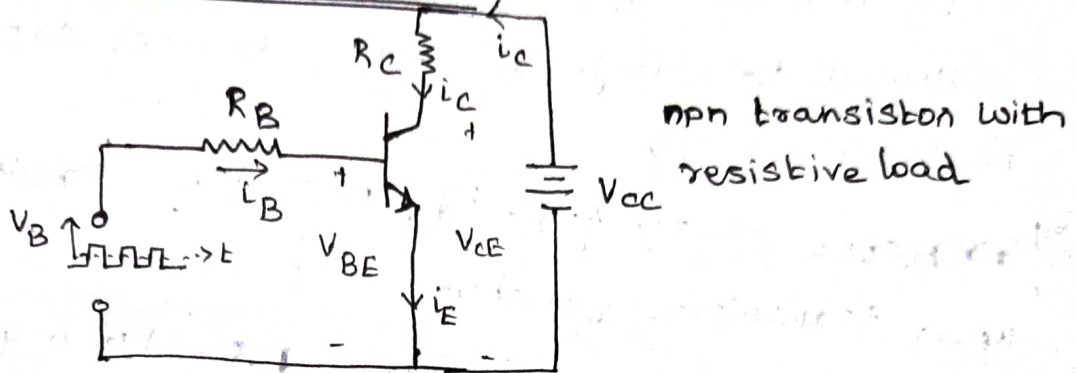
$$\beta_{forced} = \frac{I_{C_s}}{I_B}$$

The total power loss in the two function is,

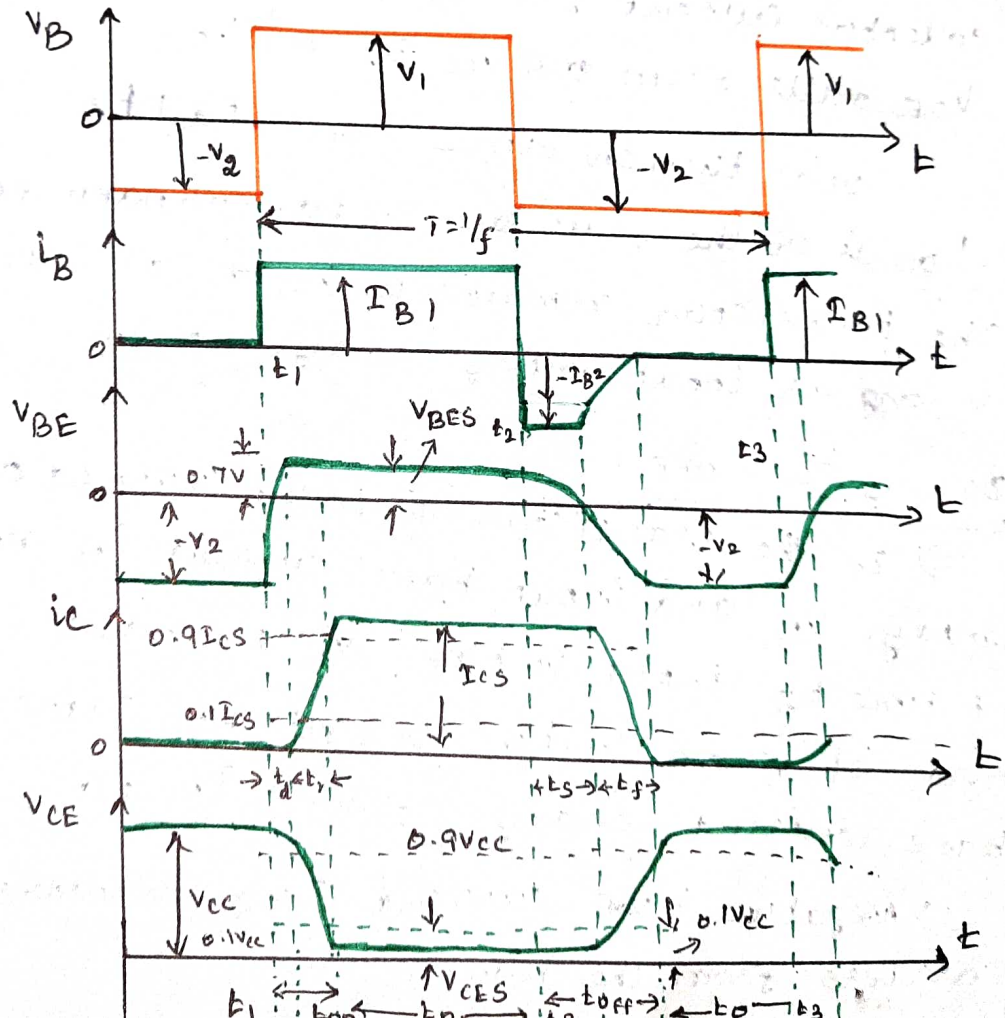
$$P_T = V_{BE} I_B + V_{CE} I_C$$

V_{BE} increases due to increased base current, resulting in increased power loss in the BEJ.

SWITCHING CHARACTERISTICS



SWITCHING WAVEFORMS



During Turn ON:

⇒ At time t_1 , input voltage V_B is made $+V_1$, and i_B rises to I_{B1} .

⇒ After t_1 , base-emitter voltage V_{BE} begins to rise gradually from $-V_2$, collector current I_C begins to rise from zero. collector-emitter voltage V_{CE} starts falling from its initial value V_{CC} .

⇒ After some time delay t_d called delay time, the collector current rises to $0.1 I_{CS}$, V_{CE} falls from V_{CC} to $0.9 V_{CC}$.

V_{BE} reaches $V_{BEs} = 0.7V$. This delay time is required to charge the base-emitter capacitance to $V_{BEs} = 0.7V$.

⇒ After delay time t_d , rise time t_r which depends upon transistor junction capacitances. At t_r , collector current rises from $0.1 I_{CS}$ to $0.9 I_{CS}$, V_{CE} falls from $0.9 V_{CC}$ to $0.1 V_{CC}$.

$$\text{Total turn on time } t_{on} = t_d + t_r$$

t_{on} is of the order of 30 to 300 nanoseconds.

The transistor remains in the on, or saturated state so long as input voltage stays at V_1 .

During Turn OFF :-

⇒ If the transistor is to be turned off, input voltage V_B and input base current i_B are reversed. At time t_2 , input voltage V_B to base circuit is reversed from V_1 to $-V_2$. At the same time, base current changes from I_{B1} to $-I_{B2}$.

⇒ Negative base current I_{B2} removes excess carriers from the base.

POWER MOSFETS

A Power MOSFET has three terminals called Drain (D), Source (S) and gate (G) in place of the corresponding three terminals Collector, emitter and Base for BJT. The circuit symbol of Power MOSFET, the arrow indicates the direction of electron flow. A Power MOSFET is a voltage-controlled device and unipolar device. Its operation depends upon the flow of majority carriers only.

Power MOSFETs are of two types :

- i) n-channel enhancement MOSFET.
- ii) p-channel enhancement MOSFET.

(i) n-channel enhancement Power MOSFET :-

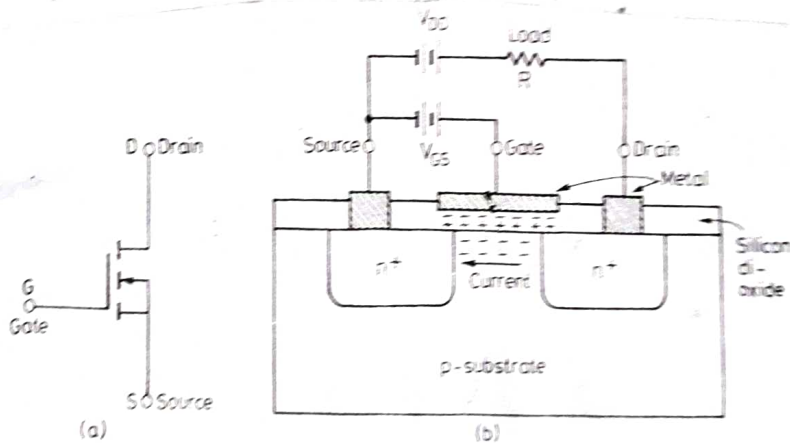
on p-substrate, two heavily doped n^+ regions are diffused. An insulating layer of silicon dioxide (SiO_2) is grown on the surface. Now this insulating layer is etched in order to embed metallic source and drain terminals. A layer of metal is also deposited on SiO_2 layer so as to form the gate of MOSFET in between source and drain terminals.

When gate circuit is open, junctions between n^+ region below drain and p-substrate is reverse biased by input voltage V_{DD} . NO current flows from drain to source and load.

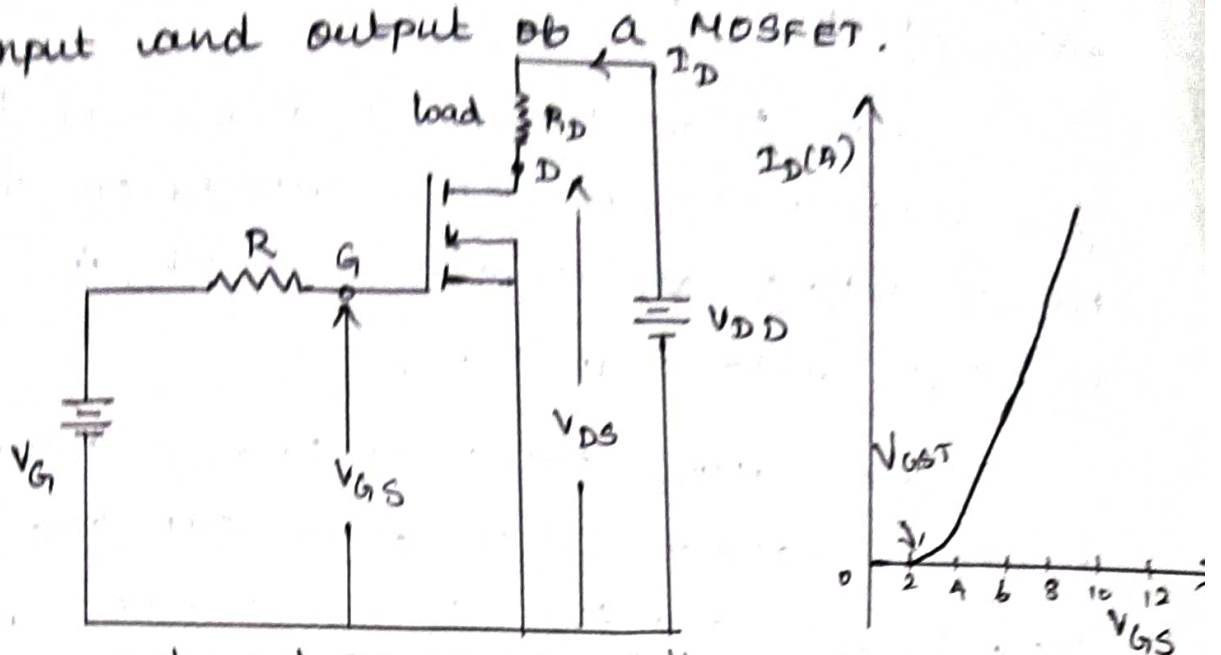
When gate is made positive with respect to source, an electric field is established.

Eventually induced negative charges in the p-substrate below SiO_2 layer are formed thus causing the p-layer below gate to become an induced n-layer. These negative charges called electrons, form n-channel between two n^+ regions and current can flow from drain to source as shown by the arrow.

If V_{GS} is made more positive, induced n-channel becomes more deep and therefore more current flows from D to S. Drain current I_D is enhanced by the gradual increase of gate voltage, hence the name enhancement MOSFET.



The basic circuit diagram for n-channel PMOSFET is shown in figure, where voltage and currents are as indicated. The source terminal S is taken as common terminal, between input and output of a MOSFET.



N channel PMOSFET circuit diagram Transfer characteristics

Transfer characteristics :
 This characteristics shows the variation of drain current I_D as a function of gate source voltage V_{GS} . V_{GST} is the minimum positive voltage between gate and source to induce n-channel. For threshold voltage below V_{GST} , device is in the off-state. Magnitude of V_{GST} is of the order of 2 to 3V.

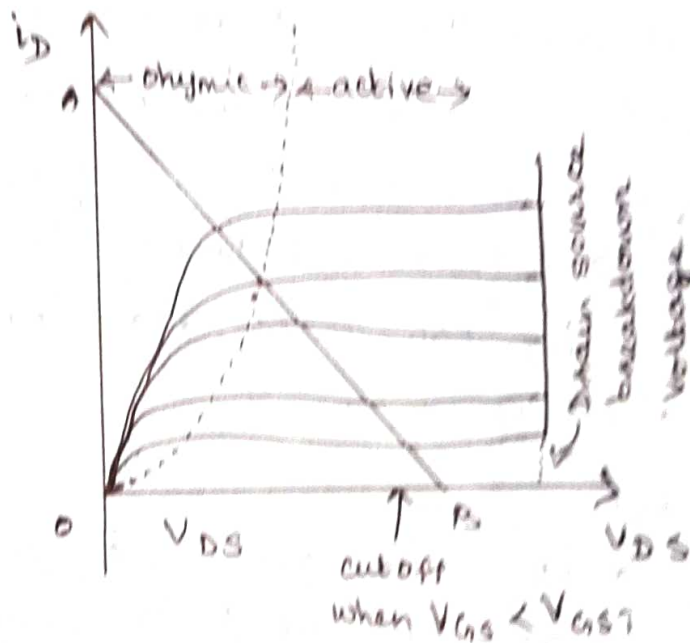
Output characteristics :

Output characteristics indicate the variation of drain current I_D as a function of drain source voltage V_{DS} . For low values of V_{DS} , the graph between $I_D - V_{DS}$ is almost linear. This indicates a constant value of on-resistance $R_{DS} = V_{DS} / I_D$. For given V_{GS} , if V_{DS} is increased,

output characteristic is relatively flat, after drain current is nearly constant. A load line intersects the output characteristics at A and B. Here A indicates fully ON condition, B indicates fully off state. PMOSFET operates as a switch either at A or B.

When Power MOSFET is driven with large gate source voltage, MOSFET is turned ON. The MOSFET acting as a closed switch, is said to be driven into ohmic region.

When device turns ON, PMOSFET traverses $i_D - V_{DS}$ characteristics from cut-off to active region and then to the ohmic region. When PMOSFET turns OFF, it takes backward journey from ohmic region to cut-off state.



output characteristics of PMOSFET.

Switching Characteristics:

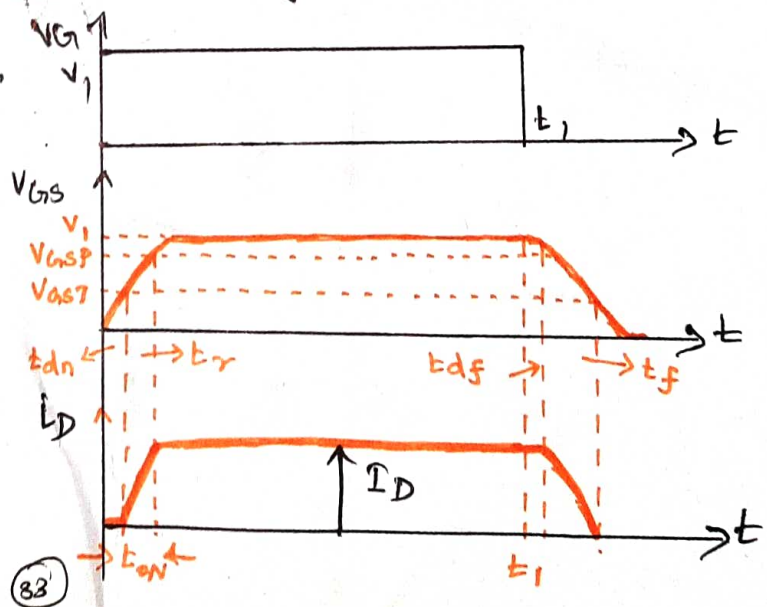
The switching characteristics of a Power MOSFET are influenced to a large extent by the internal capacitance of the device and the internal impedance of the gate drive circuit.

At turn-on, there is an initial delay t_{dn} during which input capacitance charges to gate threshold voltage V_{GS7} . Here t_{dn} is called turn-on delay time.

There is further delay, t_r called rise time, during which gate voltage rises to V_{GSP} , a voltage sufficient to drive the MOSFET into ON state. During t_r , drain current rises from zero, to full-on current I_D . The total turn on time is, $t_{on} = t_{dn} + t_r$.

Turn off process is initiated soon, after removal of gate voltage at time t_1 . The turn-off delay time, t_{df} , is the time during which input capacitance discharges from overdrive gate voltage V_1 to V_{GSP} . Fall time t_f is the time during which input capacitance discharges from V_{GSP} to threshold voltage.

Switching waveforms for PMOSFET.



Insulated Gate Bipolar Transistor (IGBT):

IGBT has been developed by combining into it the best qualities of both BJT and MOSFET. An IGBT possesses high input impedance like a PMOSFET and has low on-state power loss as in a BJT. IGBT is free from second breakdown problem.

Basic Structure :-

In IGBT, P^+ substrate is called injection layer because it injects holes into n layer. The n layer is called drift region. As in other semiconductor devices, thickness of n layer determines the voltage blocking capability of IGBT. The n layer is called body of IGBT. The n layer in between P^+ and p regions serves to accommodate the depletion layer of Pn junction. (Junction J_2).

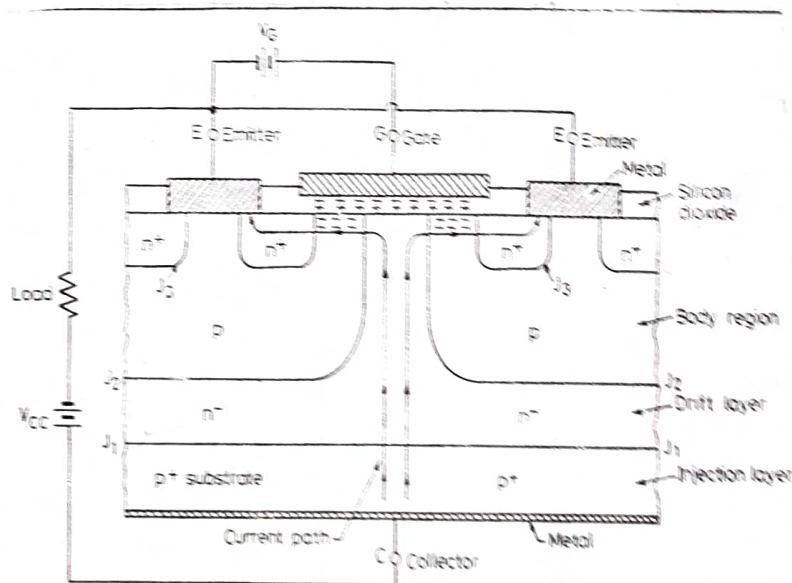


Fig. 2.19. Basic structure of an insulated gate bipolar transistor (IGBT)

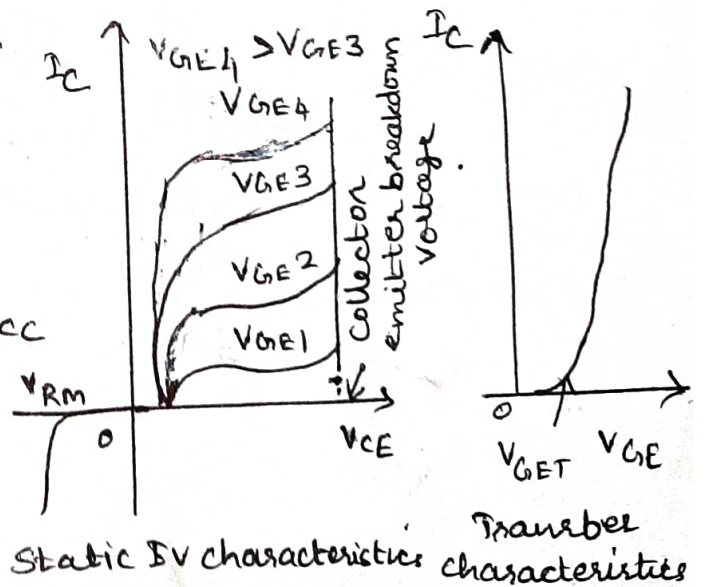
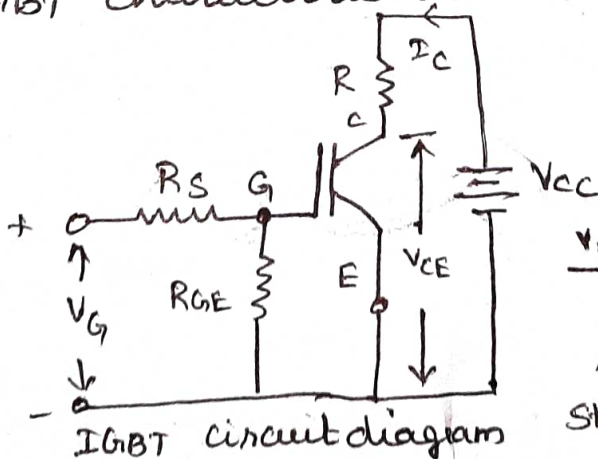
Working :

When collector is made positive with respect to emitter, IGBT gets forward biased, with no voltage between gate and emitter, two junctions between n^- region and p region are reverse biased. So, no current flows from collector to emitter.

When gate is made positive, with respect to emitter by voltage V_G , with gate-emitter voltage more than the threshold voltage V_{GET} of IGBT, an n -channel or inversion layer is formed in the upper part of p region just beneath the gate. Electrons from the n^+ emitter begin to flow to n^- drift region through n -channel.

As IGBT is forward biased, with collector positive and emitter negative, p^+ collector region injects holes into n^- drift region. In short n^- drift region is flooded with electrons from p -body region and holes from p^+ collector region. The injection carrier density in n^- drift region increases IGBT gets turned on and begins to conduct forward current I_C .

IGBT characteristics :-



The delay time is defined as the time for the collector-emitter voltage to fall from V_{CE} to $0.9 V_{CE}$. V_{CE} is the initial collector-emitter voltage. Time t_{dn} defined as, the time for the collector current to rise from its initial leakage current I_{CE} to $0.1 I_C$.

$I_C \rightarrow$ final value of collector current.

Rise time (t_r) :-

The rise time t_r is the time during which collector-emitter voltage falls from $0.9 V_{CE}$ to $0.1 V_{CE}$. It is also defined as the time for the collector current to rise from $0.1 I_C$ to its final value I_C . After time t_{on} , the collector current is I_C and the collector-emitter voltage falls to small value called conduction drop.

Turn off time :

It consists of 3 intervals

i) delay time t_{df} (ii) Initial fall time t_{f1}

iii) Final fall time t_{f2} .

$$t_{off} = t_{df} + t_{f1} + t_{f2}$$

Delay time :

The time during which gate voltage falls from V_{GE} to threshold voltage V_{GET} . As V_{GE} falls to V_{GET} during t_{df} , the collector current falls from I_C to $0.9 I_C$. At the end of t_{df} , V_{CE} begins to rise.

First fall time (t_{f1}) :

It is defined as the time during which collector current falls from 90 to 10% of its initial value I_C . (or) V_{CE} rises from V_{CES} to $0.1 V_{CE}$.

Final fall time (t_{fa}) :-

The time during which collector current falls from 20 to 10% of I_c , or the time during which collector emitter voltage rises from V_{ces} to $0.1V_{ce}$. The final fall time (t_{fa}) is the time during which collector current falls from 20 to 10% of I_c , or the time during which V_{ce} rises from $0.1V_{ce}$ to final value V_{ce} .

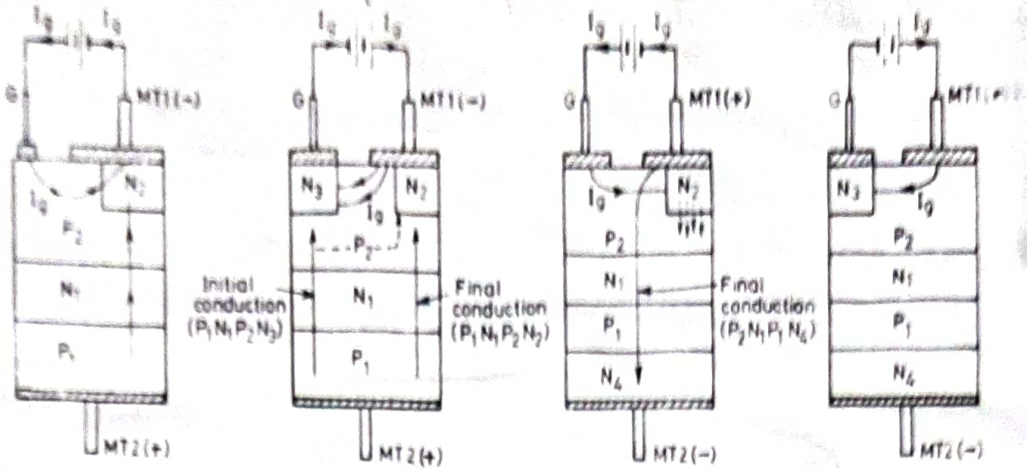
The Triac

A triac is a bidirectional thyristor with three terminals. It conducts in both the directions. When in operation, a triac is equivalent to two SCRs connected in antiparallel. The three terminals are MT₁ (main terminal 1), MT₂, and the Gate (G).

Cross sectional view of a Triac

The Gate G is near terminal MT₁. The G is connected to N_3 as well as P_2 . Terminal MT₁ is connected to P_2 and N_2 ; terminal MT₂ is connected to P_1 and N_4 .

With no signal to gate, the triac will block both half cycles of the ac applied voltage in case peak value of this voltage is less than the breakover voltage of V_{BO1} or V_{BO2} of the triac. Terminal MT₁ is taken as the point for measuring the voltage and current at the gate and MT₂ terminals.



Turn ON process of a triac:

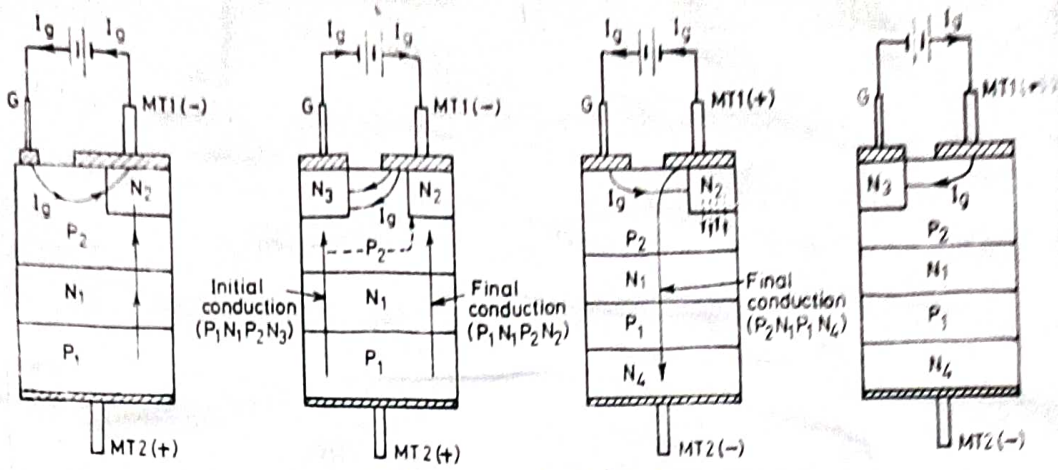
(i) MT_2 is positive and gate current is also positive:

when MT_2 is positive with respect to MT_1 , junction P_1, N_1 , P_2, N_2 are forward biased but junction N_1, P_2 is reverse biased. When gate terminal is positive with respect to MT_1 , gate current flows mainly through P_2, N_2 junction like an ordinary SCR.

When gate current has injected sufficient charge into P_2 layer, reverse biased junction N_1, P_2 breakdown as in a normal SCR. Triac starts conducting through P_1, N_1, P_2, N_2 layers. Triac operates in the first quadrant.

(ii) MT_2 is positive but gate current is negative:

When gate terminal is negative with respect to MT_1 , gate current flows through P_2, N_3 junction. Triac starts conducting through P_1, N_1, P_2, N_3 layers initially. With the conduction of P_1, N_1, P_2, N_3 , the voltage drop across this path falls but potential of layer between P_2, N_3 rises towards the anode potential of MT_2 . Left hand region being at higher potential than its right hand region. Right hand part of triac consisting of main structure P_1, N_1, P_2, N_2 begins to conduct.



Turn on process of a triac:

(i) MT₂ is positive and gate current is also positive:
 when MT₂ is positive with respect to MT₁, junction P₁N₁, P₂N₂ are forward biased but junction N₁P₂ is reverse biased. When gate terminal is positive with respect to MT₁, gate current flows mainly through P₂N₂ junction like an ordinary SCR.

When gate current has injected sufficient charge into P₂ layer, reverse biased junction N₁P₂ breakdowns as in a normal SCR. Triac starts conducting through P₁N₁P₂N₂ layers. Triac operates in the first quadrant.

(ii) MT₂ is positive but gate current is negative:
 when gate terminal is negative with respect to MT₁, gate current flows through P₂N₃ junction. Triac starts conducting through P₁N₁P₂N₃ layers initially. With the conduction of P₁N₁P₂N₃, the voltage drop across this path falls but potential of layer between P₂N₃ rises towards the anode potential of MT₂. Left hand region being at higher potential than its right hand region. Right hand part of triac consisting of main structure P₁N₁P₂N₂ begins to conduct.

(iii) MT_2 is negative but gate current is positive:

The gate current I_g forward biases P_2N_2 junction. Layer N_2 injects electrons into P_2 layer. Reverse biased junction N_1P_1 breaks down. The structure $P_2N_1P_1N_4$ is completely turned on. The triac is turned on by remote gate N_2 , the device is less sensitive in the third quadrant with positive gate current.

(iv) Both MT_2 and gate current are negative :-

The gate current I_g flows from P_2 to N_3 . Reverse biased junction N_1P_1 is broken and finally the structure $P_2N_1P_1N_4$ is turned on completely.

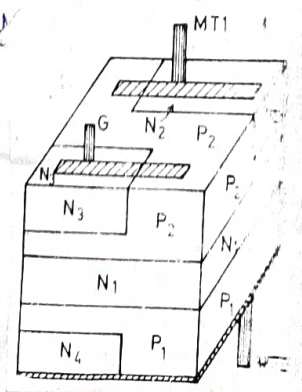
Triac with voltage and current ratings of 1200V and 300 A (rms) are available.

A triac operate in the rectifier mode than in the bidirectional mode, due to following reasons:

(a) For a given value of +ve gate current, a triac may turn on with MT_2 (+ve) in first quadrant but fail to turn on with MT_2 (-ve).

(b) With constant negative gate current, the triac may turn on with MT_2 (-ve) in third quadrant but may not turn on with MT_2 (+ve).

The rectifier mode can be overcome by increasing I_g .



Static I-V characteristics :-

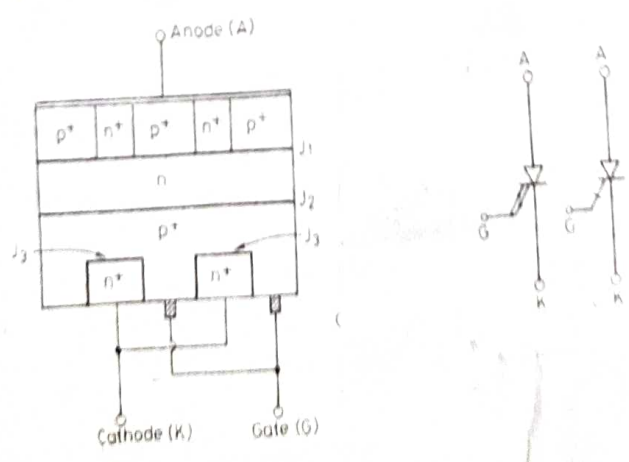
A GTO is a pnpn device that can be turned on by a positive gate current and turned off by a negative gate current at its gate cathode terminals.

Basic structure :-

A GTO is pnpn, three terminal device with anode (A), Cathode (K) and gate (G). The 4 layers are $p^+ n p^+ n^+$. It can be modelled by two-transistor analogy. Transistor Q_1 is $p^+ n p^+$ type and transistor Q_2 is $n p^+ n^+$ type, with p^+ emitter of Q_1 as anode A and n^+ emitter of Q_2 as cathode K.

Static I-V characteristics :-

Latching current for GTO is several amperes say 2A, compared to 100-500mA. If gate current is not able to turn on the GTO, it behaves like a high-voltage, low gain transistor. In the reverse mode, reverse blocking capability of GTO is low, say 20 to 30 V due to (i) anode shorts (ii) Large doping densities of both sides of reverse blocking junction J_3 .



Switching Performance :

For turning on a GTO, first triac TRI is turned on, this in turn switches on TR₂ to apply a positive gate-current pulse to turn on GTO. For turning off the GTO, the turn off circuit should be capable of outputting a high peak current.

Turn off process is initiated by gating Thyristor T₂. When T₂ is turned on, a large negative gate current pulse turns off the GTO.

Gate turn-on :-

Gate turn on for GTO is made up of delay time, rise time and spread time. Turn on time in a GTO can be decreased by increasing its forward gate current as in a thyristor.

A steep-fronted gate pulse is applied to turn on GTO. Gate drive can be removed once anode current exceeds latching current. Even after GTO is on, a continuous gate current called back porch current, I_{gb} should be applied during the entire on period of GTO. It is to avoid unwanted turn off of the GTO.

Gate turn-off :-

The total turn-off time t_q is subdivided into three different periods

i) storage period (t_s) (ii) Fall period (t_f)

iii) Tail period (t_t).

$$t_q = t_s + t_f + t_t$$

⇒ Turn off process starts as soon as negative gate current begins to flow after $t=0$, at instant A.

⇒ During the storage period, anode current I_a and anode voltage remain constant. Termination of the storage period is indicated by a fall in I_a and rise in V_a .

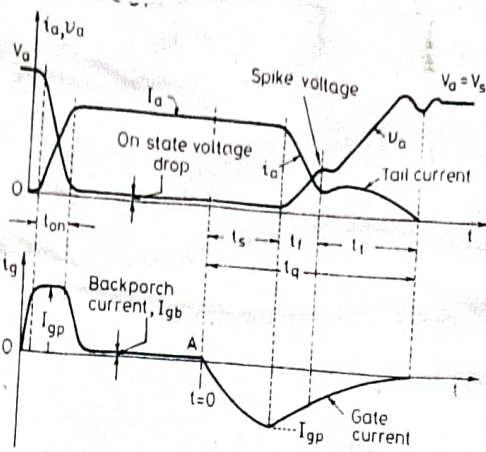
⇒ During t_s , excess charges i.e. holes in p^+ base are removed by negative gate current.

⇒ After t_s , anode current begins to fall rapidly and anode voltage starts rising. The interval during which anode current falls rapidly, is the fall time (t_f), and is the order of 1 μ sec.

⇒ The fall current measured from the instant gate current is maximum negative to the instant anode current falls to its tail current. At the time $t = t_s + t_f$.

⇒ After t_f , anode current i_a and anode voltage V_a keep moving towards their turn-off values for a time t_t called tail time.

⇒ After t_t , anode current reaches zero value, V_a undergoes a transient overshoot due to the presence of R_s, C_s . The turn off process is complete when tail current reaches zero.



GTO - switching characteristics.

Triggering Methods :

With anode positive with respect to cathode, a thyristor can be turned on by any one of the following techniques :

- (a) Forward voltage triggering
- (b) Gate triggering
- (c) dv/dt triggering
- (d) Temperature triggering.
- (e) Light triggering.

(a) Forward voltage triggering :-

When forward voltage is applied between anode and cathode, with gate circuit open, junction J_2 is reverse biased. Depletion layer is formed across junction J_2 . The width of this layer decreases with an increase in anode-cathode voltage. If forward voltage across anode-cathode is gradually increased, the depletion layer across J_2 vanishes.

Reverse biased junction J_2 is said to have avalanche breakdown and the voltage at which it occurs is called forward breakover voltage V_{BO} . At this voltage thyristor changes from off state to on-state. Junctions J_1 and J_3 are already forward biased, breakdown of Junction J_2 allows free movement of carriers across 3 junctions, large forward anode-current flows. This is limited by the load impedance. Forward breakover voltage is taken as the final voltage rating of the device during the design of SCR applications.

After the avalanche breakdown, Junction J_2 loses its reverse blocking capability. If the anode voltage is reduced below V_{BO} , SCR will continue conduction of the current.

(b) Gate Triggering :

It is a simple, reliable and efficient, the most useful method of biasing the forward biased SCRs. A positive gate voltage between gate and cathode is applied. Charges are injected into the inner p layer and voltage at which forward breakover occurs is reduced. Higher the gate current, lower is the forward breakover voltage.

When positive gate current is applied, gate p-layer is flooded with electrons from the cathode. This is because n layer is heavily doped as compared to gate p layer.

As the thyristor is forward biased, some of these electrons reach Junction J_2 . Width of depletion layer near Junction J_2 is reduced. This causes the junction J_2 to breakdown at an applied voltage lower than the forward breakover voltage V_{BO} .

c) dv/dt triggering :-

With forward voltage across the anode and cathode of a thyristor, the two outer junction J_1, J_3 are forward biased, but inner junction J_2 is reverse biased. Space charges exist in the depletion region near junction J_2 , it behaves like a capacitance.

The entire suddenly applied forward voltage V_a appears across Junction J_2 , the charging current is,

$$i_c = \frac{dQ}{dt} = \frac{d}{dt} (C_j \cdot V_a)$$

$$= C_j \cdot \frac{dV_a}{dt} + V_a \cdot \frac{dC_j}{dt}$$

Junction capacitance is almost constant,

$\frac{dC_j}{dt}$ is zero,

$$i_c = C_j \cdot \frac{dV_a}{dt}$$

$\therefore \frac{dV_a}{dt}$ is high, the charging current $i_c \uparrow$.

cd) Temperature triggering :

During forward blocking, most of the applied voltage appears across reverse biased

J_3 , so a leakage forward current always associated with SCR.

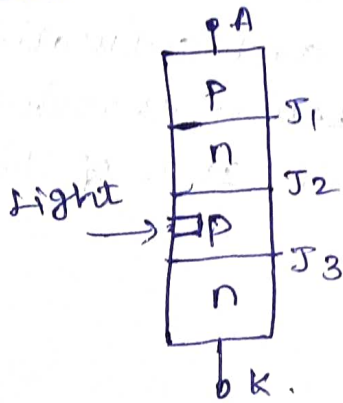
If we increase the temperature the leakage current will also increase and heat dissipation of Junction J_2 occurs.

Disadvantages :-

- 1) This type of triggering causes local hot spot and may cause thermal run away of the device.
- 2) It is very costly as protection is costly.
- 3) Triggering cannot be controlled easily.

Light Triggering :-

For light triggered SCRs, a recess (or niche) is made in the inner p layer, as shown in fig.



\Rightarrow when this recess is irradiated free charge carriers (holes & electrons) are generated.

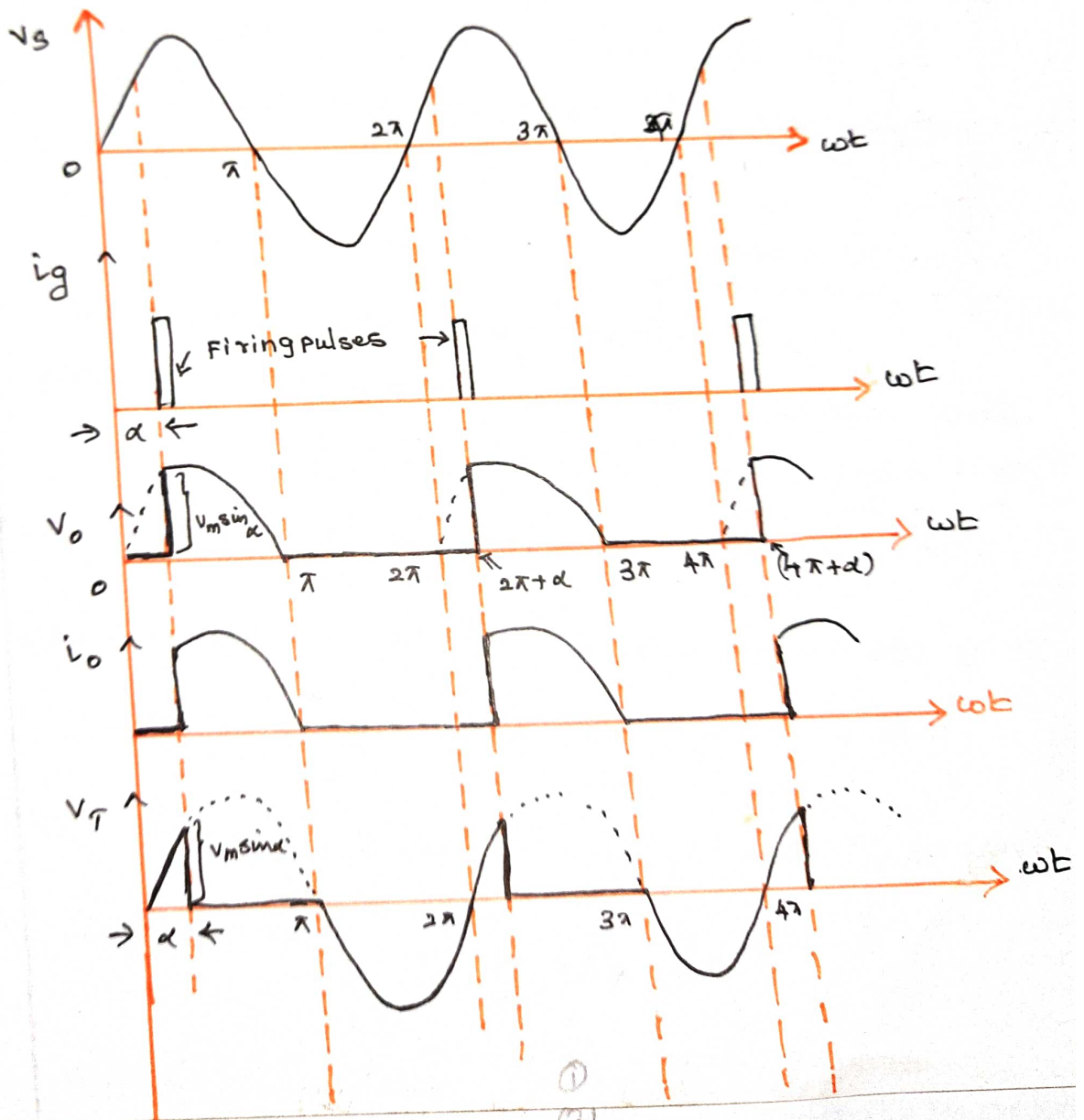
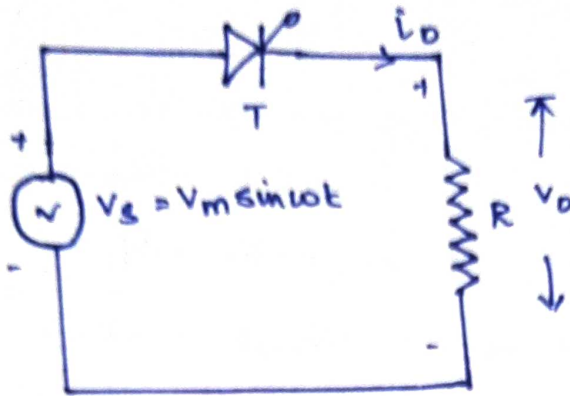
\Rightarrow If the intensity of this light thrown on the recess exceeds a certain value, forward-biased SCR is turned on.

\Rightarrow Also known as light activated SCR (LASCR).

PHASE-CONTROLLED CONVERTERS

2-pulse, 3-pulse and 6-pulse converters - performance Parameters - Effect of source inductance - Gate circuit schemes for phase control - dual converters.

Single phase half-wave thyristor circuit with R Load: $\leftarrow V_1 \rightarrow$



circuit operation :-

An SCR can conduct only when anode voltage is positive and a gating signal is applied. At some delay angle α , a positive gate signal applied between gate and cathode turns ON the SCR. Full supply voltage is applied to the load as V_o .

Firing angle :-
~.~.~.~.~.~.

A firing angle may be defined as the angle measured from the instant SCR gets forward biased to the instant it is triggered.

once the SCR is ON, load current flows, it is turned-off by reversal of voltage. At $\omega t = \pi, 3\pi, 5\pi$ etc. load current falls to zero, supply voltage reverse biases the SCR, the device is turned off.

phase control :-
~.~.~.~.~.~.

phase relationship between the start of the load current and the supply voltage can be controlled.

Thyristor remains ON from $\omega t = \alpha$ to $\pi, (2\pi + \alpha)$ to 3π etc.

It is off from π to $(2\pi + \alpha)$, 3π to $(4\pi + \alpha)$ etc.

circuit turn off time $t_c = \frac{\pi}{\omega}$ Sec.

Average voltage V_o across load R

$$V_o = \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m \cdot \sin \omega t \cdot d(\omega t).$$

$$= \frac{V_m}{2\pi} \left[-\cos \omega t \right]_{\alpha}^{\pi}$$

$$= \frac{V_m}{2\pi} \left[-\cos \pi - (-\cos \alpha) \right]$$

$$= \frac{V_m}{2\pi} \left[-(-1) + \cos \alpha \right]$$

$$\therefore \cos \pi = -1.$$

$$V_o = \frac{V_m}{2\pi} \left[1 + \cos \alpha \right]$$

$$V_{o \text{ r.m.s}} = \left[\frac{1}{2\pi} \int_{\alpha}^{\pi} V_m \sin \omega t \, d(\omega t) \right]^{1/2}$$

$$= V_m \left[\frac{1}{2\pi} \int_{\alpha}^{\pi} \sin \omega t \, d(\omega t) \right]^{1/2}$$

$$= V_m \left[\frac{1}{2\pi} \int_{\alpha}^{\pi} \frac{1 - \cos 2\omega t}{2} \, d(\omega t) \right]^{1/2}$$

$$= V_m \left[\frac{1}{2\pi \cdot 2} \int_{\alpha}^{\pi} 1 - \cos 2\omega t \, d(\omega t) \right]^{1/2}$$

$$= \frac{V_m}{2\sqrt{\pi}} \left[\left[\omega t - \frac{\sin 2\omega t}{2} \right]_{\alpha}^{\pi} \right]^{1/2}$$

$$= \frac{V_m}{2\sqrt{\pi}} \left[\pi - \frac{\sin 2\pi}{2} - \alpha + \frac{\sin 2\alpha}{2} \right]^{1/2}$$

$$\therefore \int \cos \omega t = \frac{\sin \omega t}{\omega}$$

$$V_{o \text{ r.m.s}} = \frac{V_m}{2\sqrt{\pi}} \left[\pi - \alpha + \frac{\sin 2\alpha}{2} \right]^{1/2}$$

The value of r.m.s current $I_{o \text{ r.m.s}}$ is

$$I_{o \text{ r.m.s}} = \frac{V_{o \text{ r.m.s}}}{R} = \frac{V_m}{2R\sqrt{\pi}} \left[\pi - \alpha + \frac{\sin 2\alpha}{2} \right]^{1/2}$$

Input volt amperes = rms source voltage \times
total rms line current

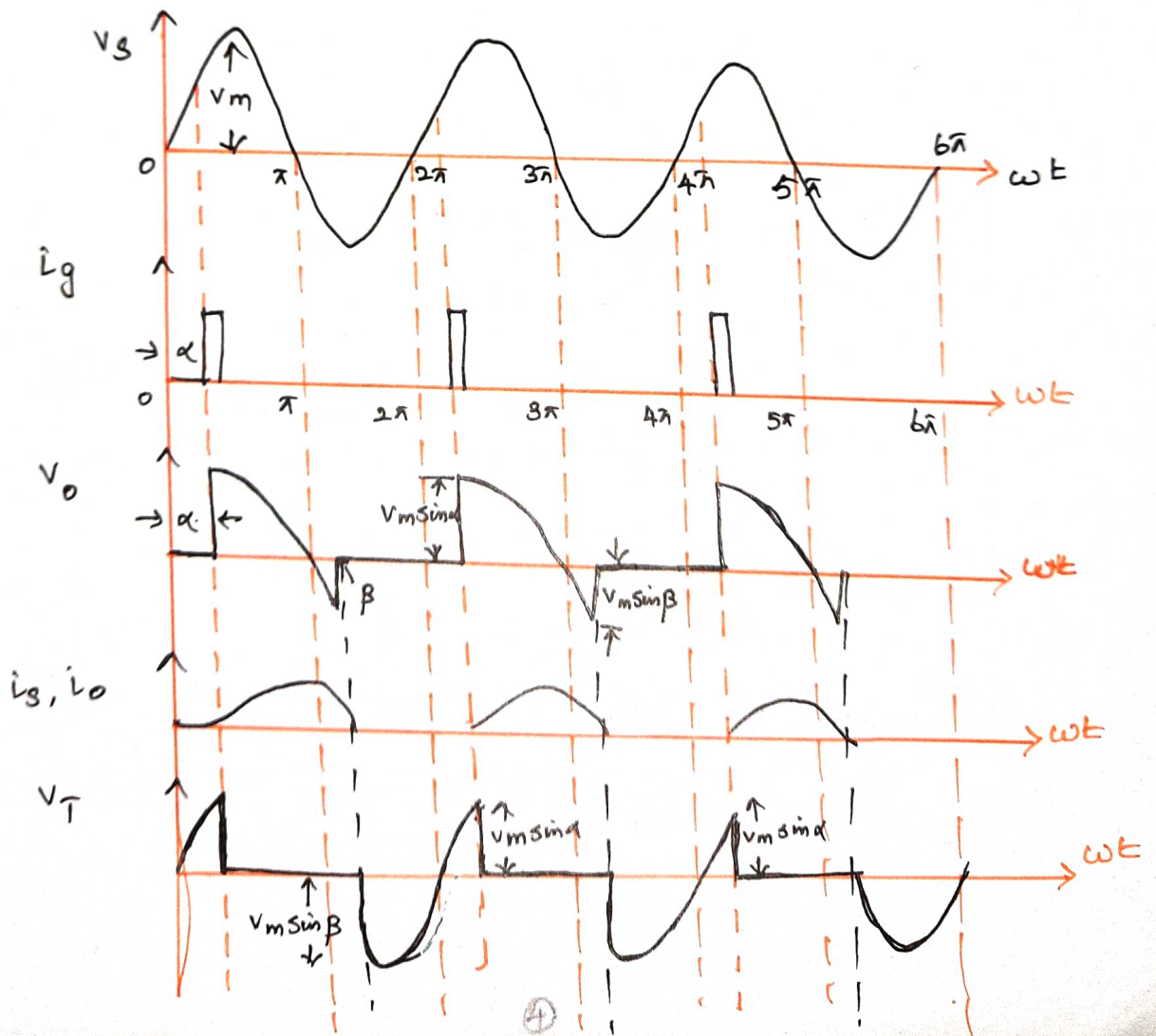
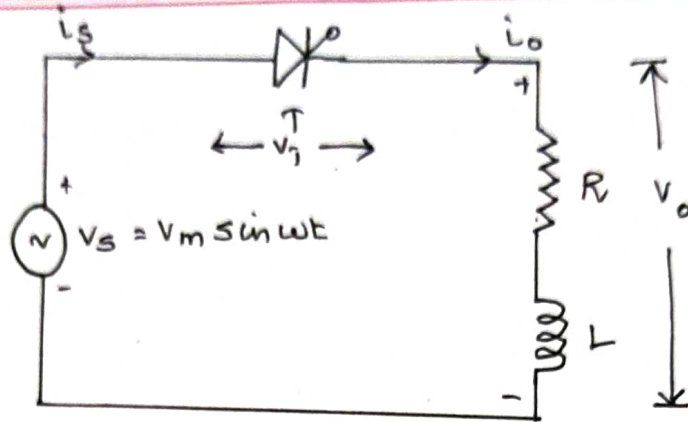
$$= V_s \cdot I_{or}$$

Input power factor = Power delivered to load

Input VA

$$= \frac{V_{or}}{V_s}$$

Single phase Half wave circuit with RL load :-



circuit operation :-

At $\omega t = \alpha$, thyristor is turned ON by gating signal. Inductance L forces the load or output current i_o to rise gradually. After some time, i_o reaches maximum value and then begins to decrease. At $\omega t = \pi$, V_o is zero, but i_o is not zero because of the load inductance.

After $\omega t = \pi$, SCR is subjected to reverse anode voltage but it will not be turned OFF as load current i_o is not less than the holding current. At some angle $\beta > \pi$, i_o reduces to zero, SCR is turned OFF as it is already reverse biased. After $\omega t = \beta$, $V_o = 0$, $i_o = 0$.

Angle β is called the extinction angle.
 $\beta - \alpha = \gamma$ is called the conduction angle.

$$\text{circuit turn-off time } t_c = \frac{2\pi - \beta}{\omega}$$

$$\text{Average load voltage } V_o = \frac{1}{2\pi} \int_{\alpha}^{\beta} V_m \sin \omega t \, d(\omega t)$$

$$= \frac{V_m}{2\pi} \left[-\cos \omega t \right]_{\alpha}^{\beta}$$

$$= \frac{V_m}{2\pi} \left[-\cos \beta + \cos \alpha \right]$$

$$V_o = \frac{V_m}{2\pi} \left[\cos \alpha - \cos \beta \right]$$

$$V_{o \text{ r.m.s}} = \left[\frac{1}{2\pi} \int_{\alpha}^{\beta} V_m \sin^2 \omega t \, d(\omega t) \right]^{1/2}$$

$$= \frac{V_m}{\sqrt{\pi}} \left[\frac{1}{2} \int_{\alpha}^{\beta} \frac{1 - \cos 2\omega t}{2} \, d(\omega t) \right]^{1/2}$$

$$\therefore \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}$$

$$= \frac{V_m}{2\sqrt{\pi}} \left\{ \left[\omega t - \frac{\sin 2\omega t}{2} \right]_{\alpha}^{\beta} \right\}^{1/2}$$

$$= \frac{V_m}{2\sqrt{\pi}} \left[\beta - \frac{\sin 2\beta}{2} - \alpha + \frac{\sin 2\alpha}{2} \right]^{1/2}$$

$$= \frac{V_m}{2\sqrt{\pi}} \left[(\beta - \alpha) - \frac{1}{2} (\sin 2\beta - \sin 2\alpha) \right]^{1/2}$$

The load current i_o consists of two components.

i) steady state component (i_s)

ii) Transient component (i_T)

$$\text{where } i_s = \frac{V_m}{\sqrt{R^2 + X^2}} \sin(\omega t - \phi)$$

$$\phi = \tan^{-1} \frac{X}{R} ; X = \omega L$$

$$i_T = A \cdot e^{-(R/L)t}$$

$$i_o = i_s + i_T = \frac{V_m}{2} \sin(\omega t - \phi) + A e^{-(R/L)t} \quad \text{--- (1)}$$

Constant A can be obtained from boundary condition at $\omega t = \alpha$.

$$t = \frac{\alpha}{\omega}, \quad i_o = 0$$

Sub these values in eqn (1)

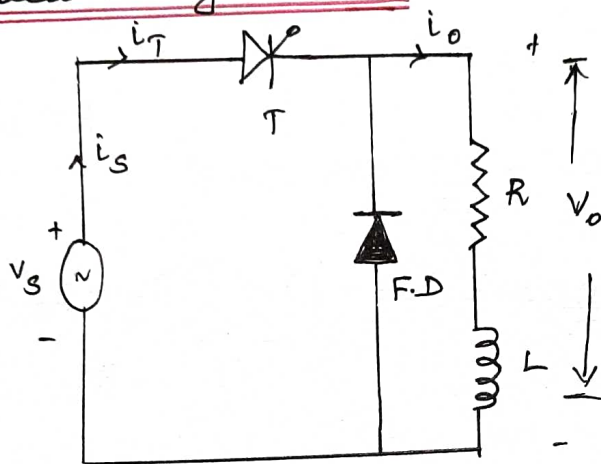
$$0 = \frac{V_m}{Z} \sin(\alpha - \phi) + A e^{-(R/L)(\alpha/\omega)}$$

$$A = -\frac{V_m}{Z} \sin(\alpha - \phi), e^{\frac{R\alpha}{\omega L}}$$

Sub the value of A in eqn ①

$$i_o = \frac{V_m}{Z} \sin(\omega t - \phi) - \frac{V_m}{Z} \sin(\alpha - \phi) \exp\left\{-\frac{R}{\omega L}(\omega t - \alpha)\right\}$$

Single phase half wave circuit with RL load and freewheeling diode:



circuit operation:-

At $\omega t = 0$, source voltage is becoming positive. At some delay angle α , forward biased SCR is triggered and source voltage V_s appears across load as V_o .

At $\omega t = \pi$, source voltage V_s is zero, freewheeling diode FD is forward biased. load current i_o is transferred from SCR to FD. It is assumed that during freewheeling period load current does not decay to zero, until the SCR is

triggered again at $(2\pi + \alpha)$.

There are two modes of operation

i) conduction mode : $\alpha \leq \omega t \leq \pi$

SCR conducts from α to π , $2\pi + \alpha$ to 3π and so on.

The voltage equation

$$V_m \sin \omega t = R i_o + L \cdot \frac{di_o}{dt}$$

$$i_o = \frac{V_m}{Z} \sin(\omega t - \phi) + A e^{-(R/L)t}$$

at $\omega t = \alpha$, $i_o = I_o$,

$$t = \alpha/\omega, i_o = I_o$$

$$A = \left[I_o - \frac{V_m}{Z} \sin(\alpha - \phi) \right] e^{R\alpha/\omega L}$$

$$i_o = \frac{V_m}{Z} \sin(\omega t - \phi) + \left[I_o - \frac{V_m}{Z} \sin(\alpha - \phi) \right] e^{-R/L(t - \frac{\alpha}{\omega})}$$

ii) Free wheeling mode : $\pi < \omega t \leq (2\pi + \alpha)$.

FD conducts from π to $2\pi + \alpha$, 3π to $4\pi + \alpha$ and so on.

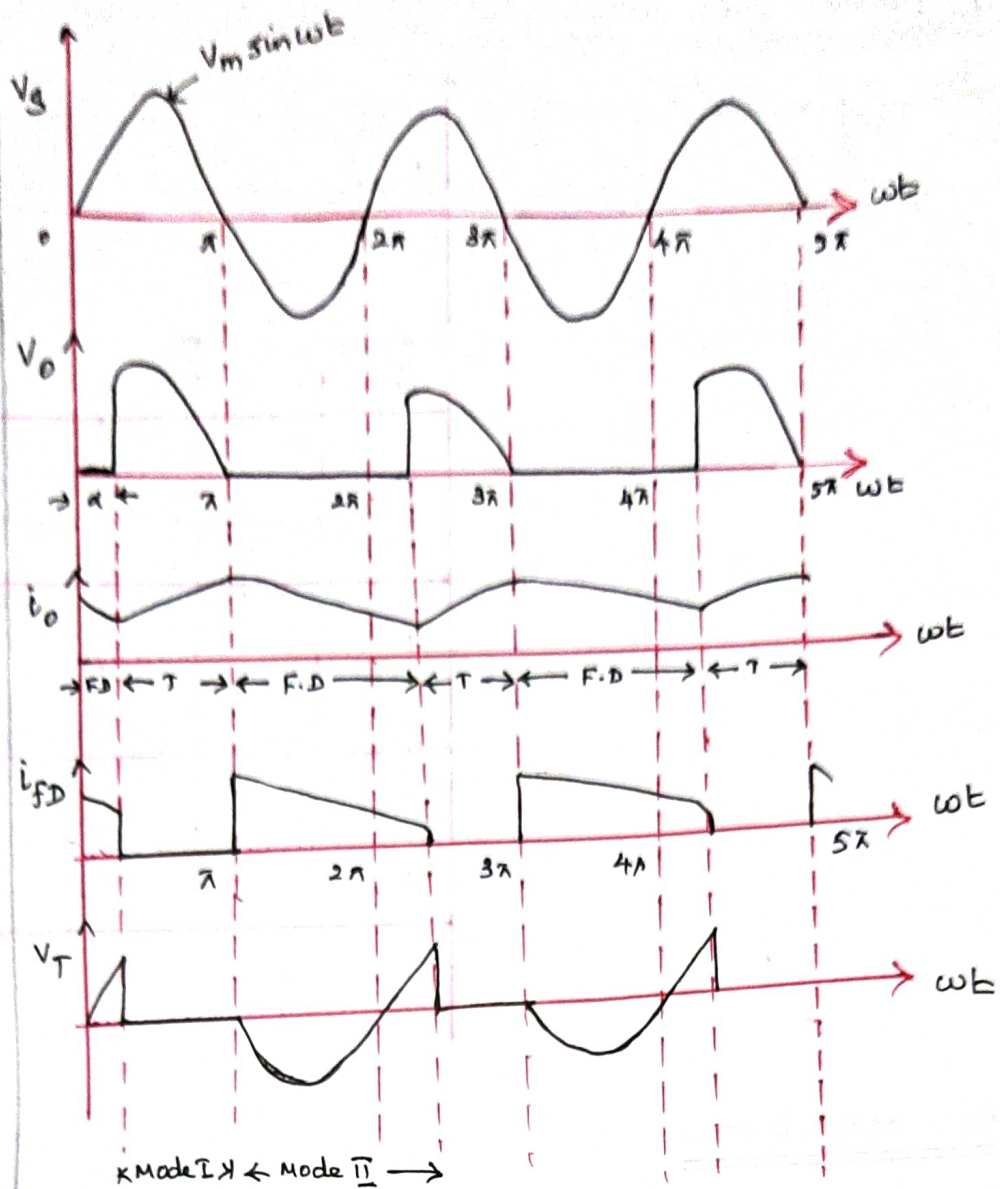
$$i_o = I_{o1} \cdot \exp \left[-R/L(t - \pi/\omega) \right]$$

average load voltage

$$V_o = \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m \sin \omega t d(\omega t)$$

$$= \frac{V_m}{2\pi} \left[-\cos \omega t \right]_{\alpha}^{\pi}$$

$$V_o = \frac{V_m}{2\pi} \left[-\cos \pi + \cos \alpha \right] = \frac{V_m}{2\pi} [1 + \cos \alpha]$$



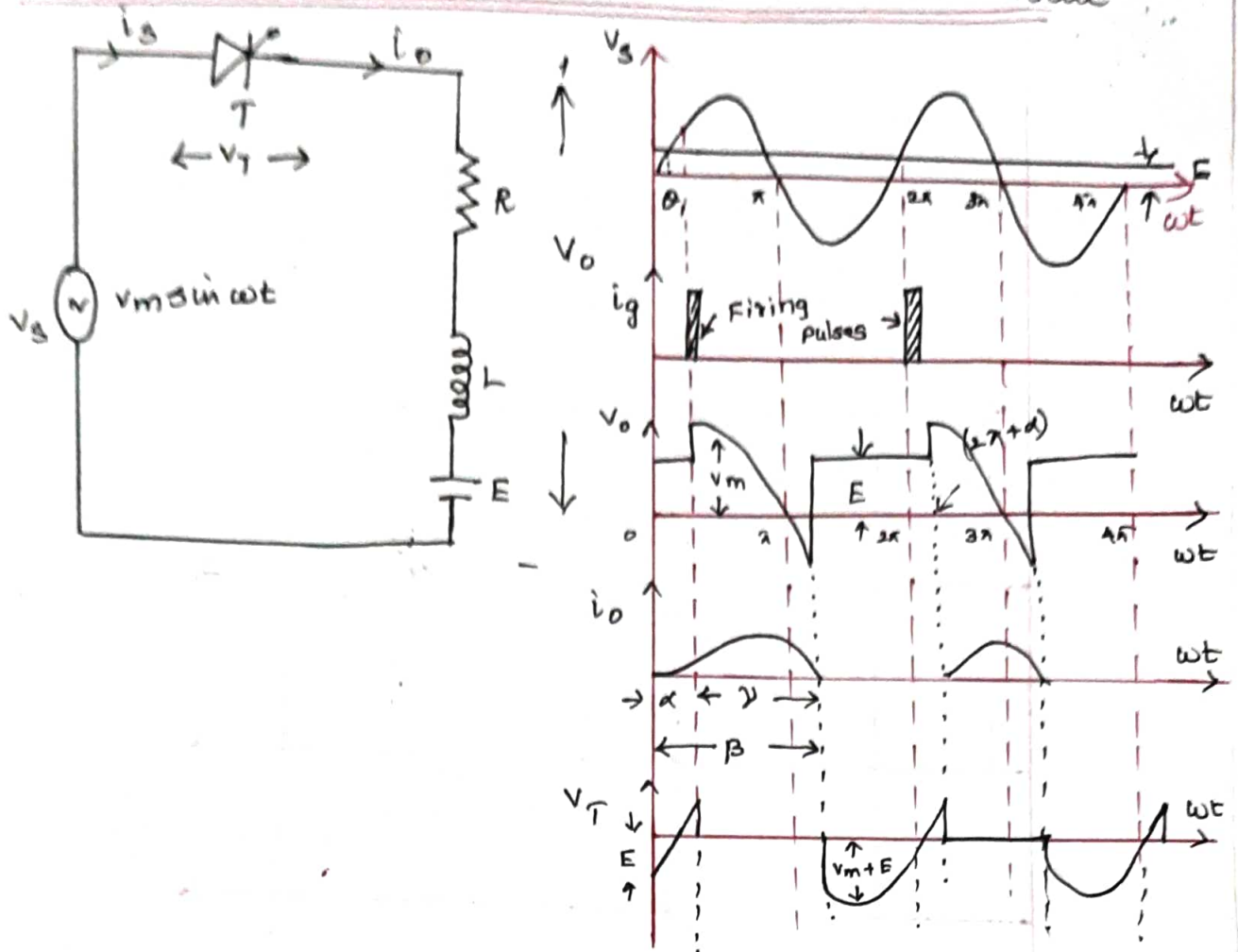
Load absorbs power for α to π

Energy stored in L is delivered to R for π to $(\alpha + \pi)$

Advantages of using freewheeling diodes are

- (i) Input power factor is improved.
- (ii) Load current waveform is improved.
- (iii) Load performance is better.
- (iv) Converter efficiency improves.

Single phase Half wave circuit with RLE load :



circuit operation :

The counter emf E in the load may be due to a battery or a d.c motor. When SCR T_1 is fired at an angle $\alpha < \theta_1$, then $E > v_s$, SCR is reverse biased. It will not turn ON. With SCR ON, the voltage equation is :

$$v_m \sin \omega t = R i_o + L \cdot \frac{di_o}{dt} + E$$

The solution of the above equation has two components:

- i) steady state current component (i_s)
- ii) Transient current component (i_T).

$$i_{s1} = \frac{V_m}{Z} \sin(\omega t - \phi)$$

$$i_{s2} = -(E/R)$$

$$i_L = A e^{-(R/L)t}$$

$$\text{Total current } i_o = i_{s1} + i_{s2} + i_L$$

$$\begin{aligned} \text{Average load voltage } V_o &= \frac{1}{2\pi} \int_{\alpha}^{\beta} (V_m \sin \omega t - E) d(\omega t) \\ &= \frac{1}{2\pi} \left[-\frac{\cos \omega t}{V_m} - E \cdot \omega t \right]_{\alpha}^{\beta} \\ &= \frac{1}{2\pi} \left[-\frac{(\cos \beta + \cos \alpha)}{V_m} - E(\beta - \alpha) \right] \end{aligned}$$

$$\text{Conduction angle } \gamma = \beta - \alpha,$$

$$\beta = \alpha + \gamma$$

$$V_o = \frac{1}{2\pi} \left[\frac{\cos \alpha - \cos(\alpha + \gamma)}{V_m} - E \gamma \right]$$

Using the trigonometric relation,

$$\cos x - \cos y = 2 \sin \frac{x+y}{2} \sin \frac{y-x}{2}$$

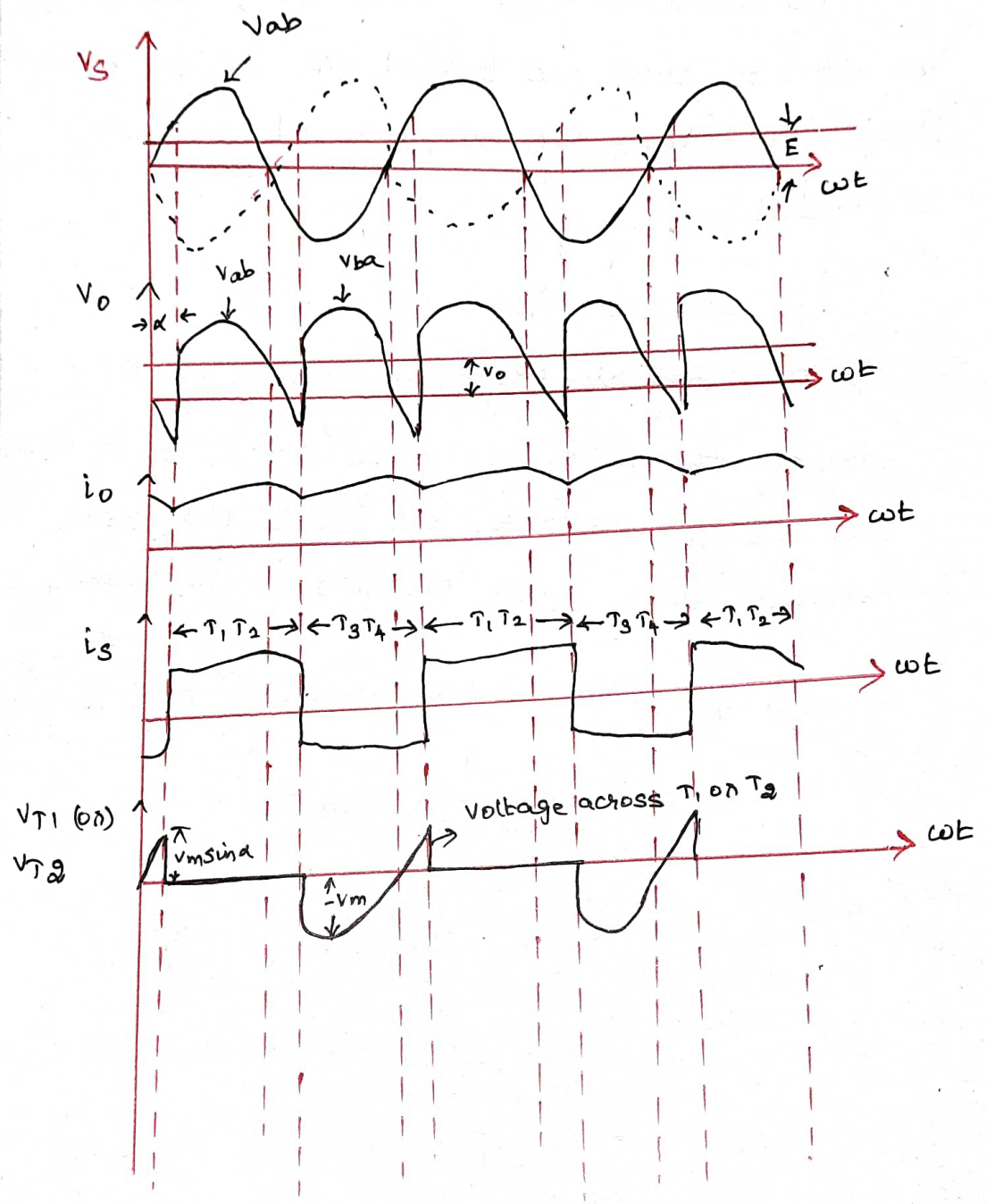
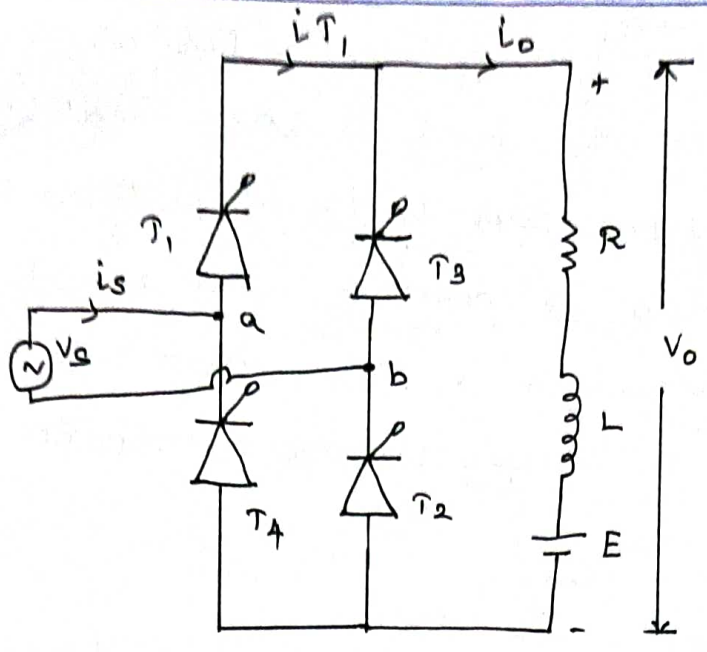
$$V_o = \frac{1}{2\pi} \left[2 \frac{V_m}{2} \sin \left[\frac{2\alpha + \gamma}{2} \right] \sin \frac{\gamma}{2} - E \cdot \gamma \right]$$

$$V_o = \frac{1}{2\pi} \left[2 V_m \sin \left(\alpha + \frac{\gamma}{2} \right) \sin \frac{\gamma}{2} - E \cdot \gamma \right]$$

$$\text{Circuit turn off time} = \frac{2\pi + \theta_1 - \beta}{\omega} \text{ sec.}$$

Single phase full converter bridge with RLE load E

[2 pulse converter]



circuit operation :-

A single phase full converter bridge is using four SCRs. The load is assumed to be of RLE type. Thyristor pair T_1, T_2 is simultaneously triggered. when a is positive with respect to B , supply voltage waveform is V_{ab} . when b is positive with respect to a , supply voltage waveform is V_{ba} .
 $V_{ab} = -V_{ba}$.

Forward biased SCRs T_1, T_2 are triggered at $\omega t = \alpha$, they get turned ON. T_3, T_4 reverse bias they are turned off by natural or line commutation.

T_1, T_2 conduct from $\omega t = \alpha$ to $\pi + \alpha$.

T_3, T_4 conduct from $\omega t = \pi + \alpha$ to $2\pi + \alpha$.

During the interval π to $(\pi + \alpha)$, V_s is negative but i_s is positive. the load therefore returns some of its energy to the supply system.

$$\text{average output voltage } (V_o) = \frac{2\pi}{2\pi} \int_{\alpha}^{\pi+\alpha} V_m \cdot \sin \omega t \cdot d(\omega t)$$

$$V_o = \frac{2V_m}{2\pi} \left[-\cos \omega t \right]_{\alpha}^{\pi+\alpha}$$

$$= \frac{V_m}{\pi} \left[-\cos(\pi + \alpha) + \cos \alpha \right]$$

$$= \frac{V_m}{\pi} \left[-[\cos \pi \cos \alpha + \sin \pi \sin \alpha] + \cos \alpha \right]$$

$$= \frac{V_m}{\pi} \left[-(-1)\cos \alpha + \cos \alpha \right]$$

$$V_o = \frac{2V_m}{\pi} \cos \alpha \quad \text{②}$$

$$\because \cos(A+B) = \cos A \cos B + \sin A \sin B$$

$$\sin \pi = 0$$

$$\cos \pi = -1$$

$$V_o \text{ r.m.s} = \left[\frac{2 \times 1}{2\pi} \int_{\alpha}^{\pi+\alpha} V_m \sin \omega t \, d(\omega t) \right]^{1/2}$$

$$= \frac{V_m}{\sqrt{\pi}} \left[\int_{\alpha}^{\pi+\alpha} \frac{1 - \cos 2\omega t}{2} \, d(\omega t) \right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2\pi}} \left[\omega t - \frac{\sin 2\omega t}{2} \right]_{\alpha}^{\pi+\alpha} \Bigg|^{1/2}$$

$$= \frac{V_m}{\sqrt{2\pi}} \left[\pi + \alpha - \frac{\sin 2(\pi + \alpha)}{2} - \alpha + \frac{\sin 2\alpha}{2} \right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2\pi}} \left[\pi + \alpha - \frac{\sin(2\pi + 2\alpha)}{2} - \alpha + \frac{\sin 2\alpha}{2} \right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2}\sqrt{\pi}} \left[\pi - \frac{(\sin 2\pi \cos 2\alpha + \cos 2\pi \sin 2\alpha)}{2} + \frac{\sin 2\alpha}{2} \right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2}\sqrt{\pi}} \left[\pi - \frac{\sin 2\alpha}{2} + \frac{\sin 2\alpha}{2} \right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2}\sqrt{\pi}} \times \sqrt{\pi} = \frac{V_m}{\sqrt{2}}$$

$$\begin{aligned} \sin(A+B) &= \\ \sin A \cos B + \\ \cos A \sin B \\ \sin 2\pi &= 0. \\ \cos 2\pi &= 1. \end{aligned}$$

Rectification mode :-

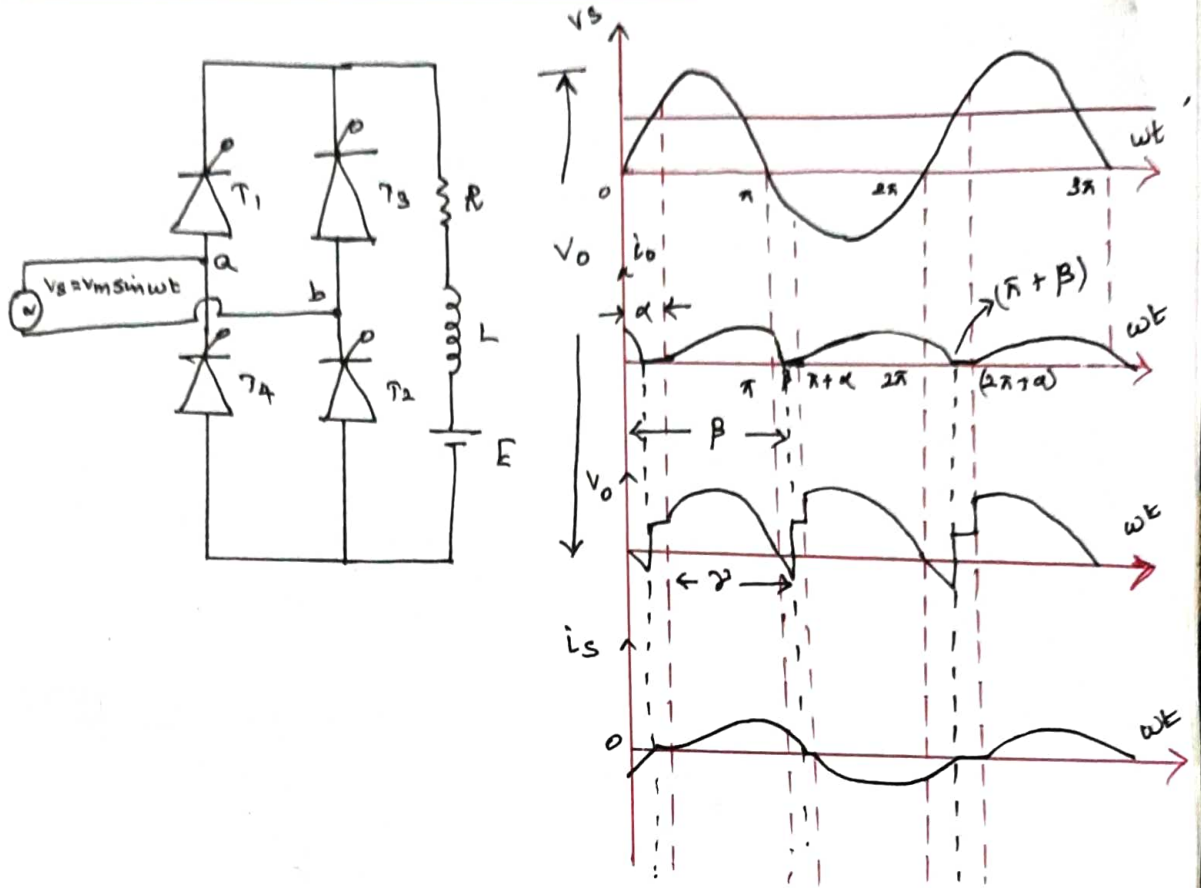
During the period from α to π , input voltage V_s , input current is are positive. Power flows from the supply to the load. The converter is said to be operated in Rectification mode.

Inversion mode :-

During the period from π to $\pi + \alpha$, input voltage V_s is negative, input current is positive.

power flows from the load to the supply.
 The converter is said to be operated in
 inversion mode.

single phase Full converter with Discontinuous current:-



In practice, the output current may become discontinuous at high values of firing angle or at low values of load current.

Discontinuous means load current reaches zero, during each half cycle before the next SCR in sequence is fired.

Continuous means load current never ceases but continuous to flow through SCR/diode.

Circuit operation :

SCR pair T_1, T_2 is triggered at $\omega t = \alpha$, load current decays to zero, here $\beta > \pi$.

T_1, T_2 are reverse biased after $\omega t = \pi$, this pair is commutated at $\omega t = \beta$ when $I_o \neq 0$.

Conduction period

$\alpha < \omega t < \beta$, T_1, T_2 conduct, $V_o = V_s$

$(\pi + \alpha) < \omega t < (\pi + \beta)$, T_3, T_4 conduct, $V_o = V_s$.

Idle period :

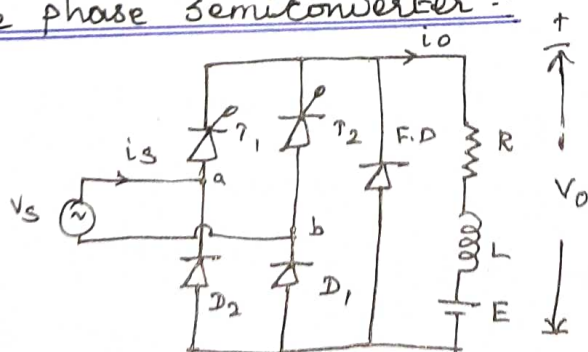
$\beta < \omega t < (\pi + \alpha)$, no circuit element conducts and $V_o = E$.

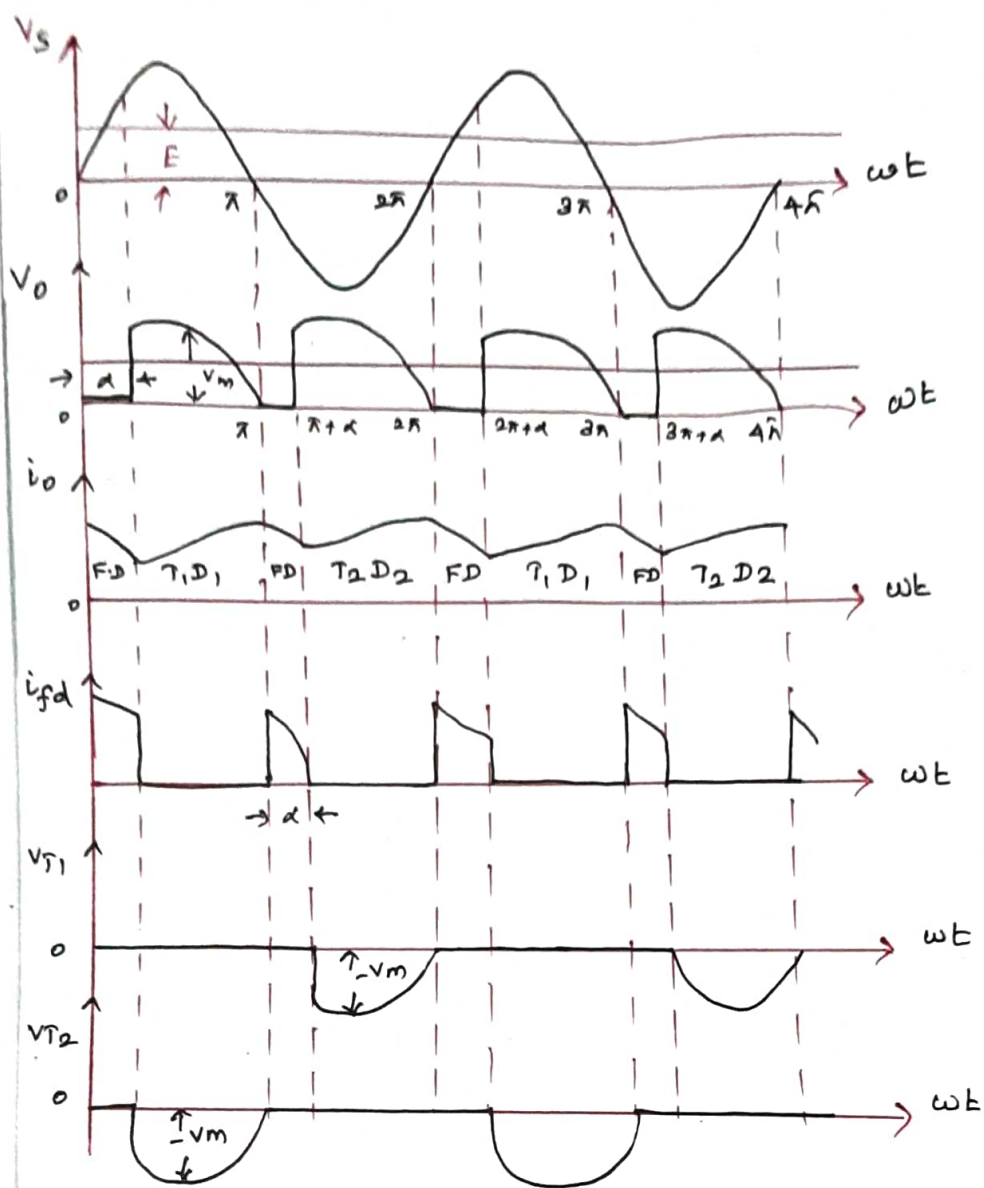
$$\begin{aligned} \text{Average load voltage } V_o &= \frac{1}{\pi} \int_{\alpha}^{\beta} V_m \sin \omega t \cdot d(\omega t) \\ &\quad + E (\pi + \alpha - \beta) \\ &= \frac{V_m}{\pi} \left[-\cos \omega t \right]_{\alpha}^{\beta} + E (\pi + \alpha - \beta) \\ &= \frac{V_m}{\pi} \left[-\cos \beta + \cos \alpha \right] + E (\pi + \alpha - \beta) \\ &= \frac{V_m}{\pi} \left[\cos \alpha - \cos \beta \right] + E (\pi + \alpha - \beta) \end{aligned}$$

where $\gamma = \beta - \alpha =$ ~~conduction angle~~.

$$= \frac{V_m}{\pi} \left[\cos \alpha - \cos \beta \right] + E$$

Single phase semiconverter :-





Circuit operation:

T_1 is triggered at $\omega t = \alpha$, $V_m \sin \alpha > E$.
 load gets connected to source through T_1, D_1 .

$\omega t = \alpha$ to π , load current i_o flows through RLE,
 D_1 , source and T_1 .

$\omega t = \pi +$, F.D gets forward biased and starts
 conducting.

$\omega t = \pi + \alpha$, T_2 will triggered, F.D reverse biased.

From $\omega t = \alpha$ to π , T_1, D_1 conducts, $i_s = +ve$.

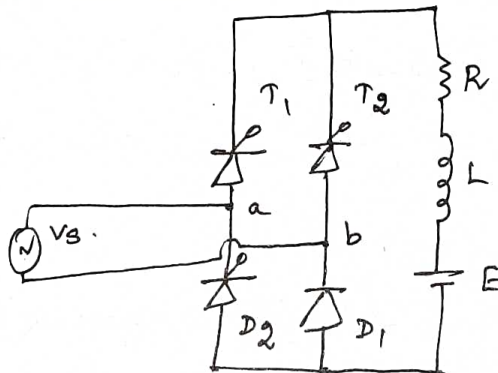
$\omega t = \pi + \alpha$ to 2π , T_2, D_2 conducts $i_s = -ve$.

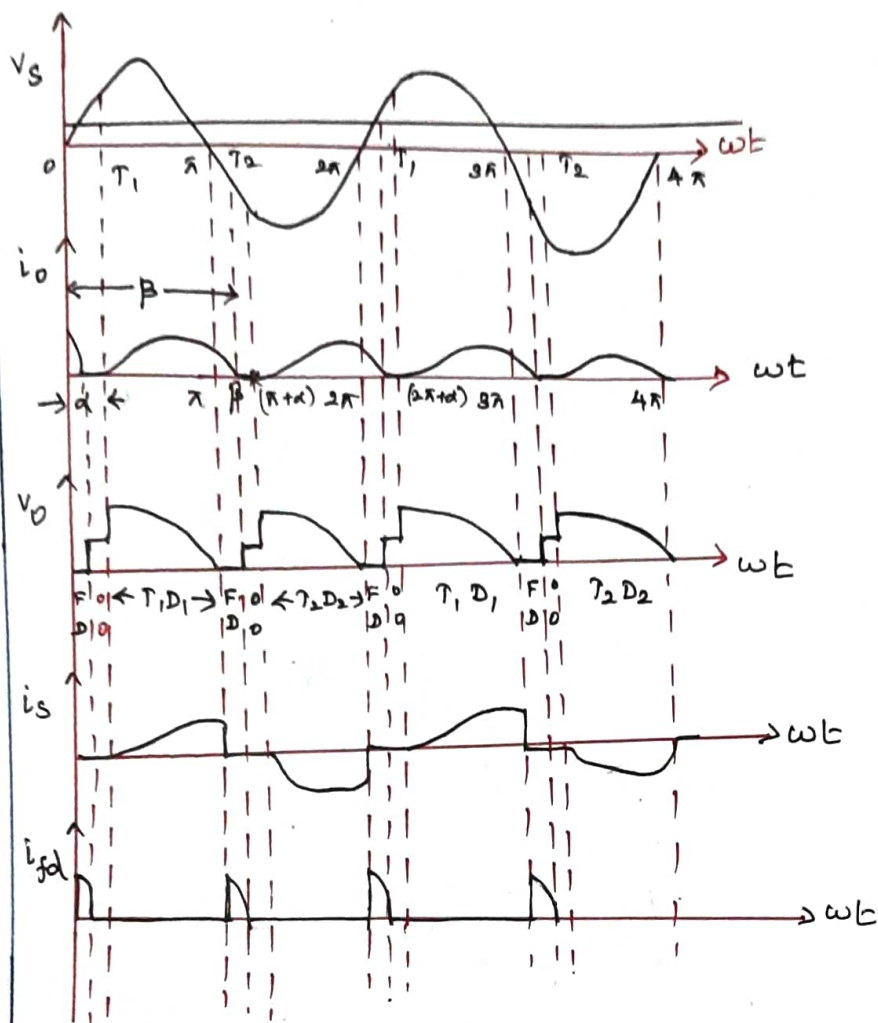
$\omega t = \pi$ to $\pi + \alpha$, \rightarrow freewheeling period.

$$\begin{aligned}
 V_o &= \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \cdot \sin \omega t \cdot d(\omega t) \\
 &= \frac{V_m}{\pi} \left[-\cos \omega t \right]_{\alpha}^{\pi} \\
 &= \frac{V_m}{\pi} \left[-\cos \pi + \cos \alpha \right] \\
 &= \frac{V_m}{\pi} \left[1 + \cos \alpha \right]
 \end{aligned}$$

$$\begin{aligned}
 V_{o, r.m.s} &= \left[\frac{1}{\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t \cdot d(\omega t) \right]^{1/2} \\
 &= \frac{V_m}{\sqrt{\pi}} \left[\int_{\alpha}^{\pi} \frac{1 - \cos 2\omega t}{2} \right]^{1/2} \\
 &= \frac{V_m}{\sqrt{2\pi}} \left[\omega t - \frac{\sin 2\omega t}{2} \right]_{\alpha}^{\pi} \\
 &= \frac{V_m}{\sqrt{2\pi}} \left[\pi - \frac{\sin 2\pi}{2} - \alpha + \frac{\sin 2\alpha}{2} \right]^{1/2} \\
 &= \frac{V_m}{\sqrt{2\pi}} \left[\pi - \alpha + \frac{\sin 2\alpha}{2} \right]^{1/2}
 \end{aligned}$$

1 ϕ semiconverter with discontinuous current :-





Conduction period :

$\alpha < \omega t < \pi$, T_1, D_1 conduct, $V_o = V_s$.

$\pi + \alpha < \omega t < 2\pi$, T_2, D_2 conduct, $V_o = V_s$.

Freewheeling period :

$\pi < \omega t < \beta$, FD conducts, $i_{fd} = i_o$, $V_o = 0$.

$2\pi < \omega t < \pi + \beta$, FD conducts, $i_{fd} = i_o$, $V_o = 0$.

Idle period :-

$\beta < \omega t < \pi + \alpha$, no circuit component conducts.

$i_o = 0$, $V_o = E$.

Average output voltage $V_o = \frac{1}{\pi} \int_{\alpha}^{\beta} V_m \sin \omega t \cdot d(\omega t) + E(\pi + \alpha - \beta)$.

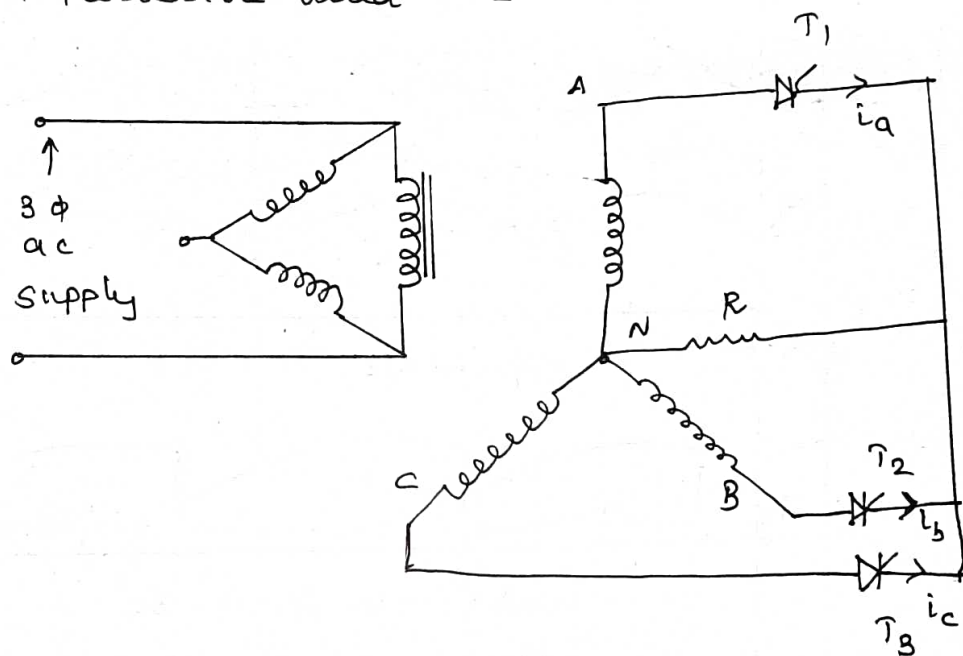
$= \frac{V_m}{\pi} (1 + \cos \alpha) + E(\pi + \alpha - \beta)$.

Three Phase Controlled Converters :- [3 pulse converter]

The converter operating from a 1ϕ supply produces high a.c ripple voltage at its d.c terminals. Smoothing reactor is necessary to smoothen the output voltage and reduce the possibility of discontinuous operation.

Higher the pulse number, smoother is the output voltage. High voltages are suitably stepped down using transformers. These transformers are delta connected on primary side and star connected on the secondary side.

Three phase Half wave controlled Rectifier with Resistive load :- [Mid point configuration]



Circuit operation :-
No SCR can be triggered below a phase angle of 30° , because it remains reverse biased by the other conducting phase.

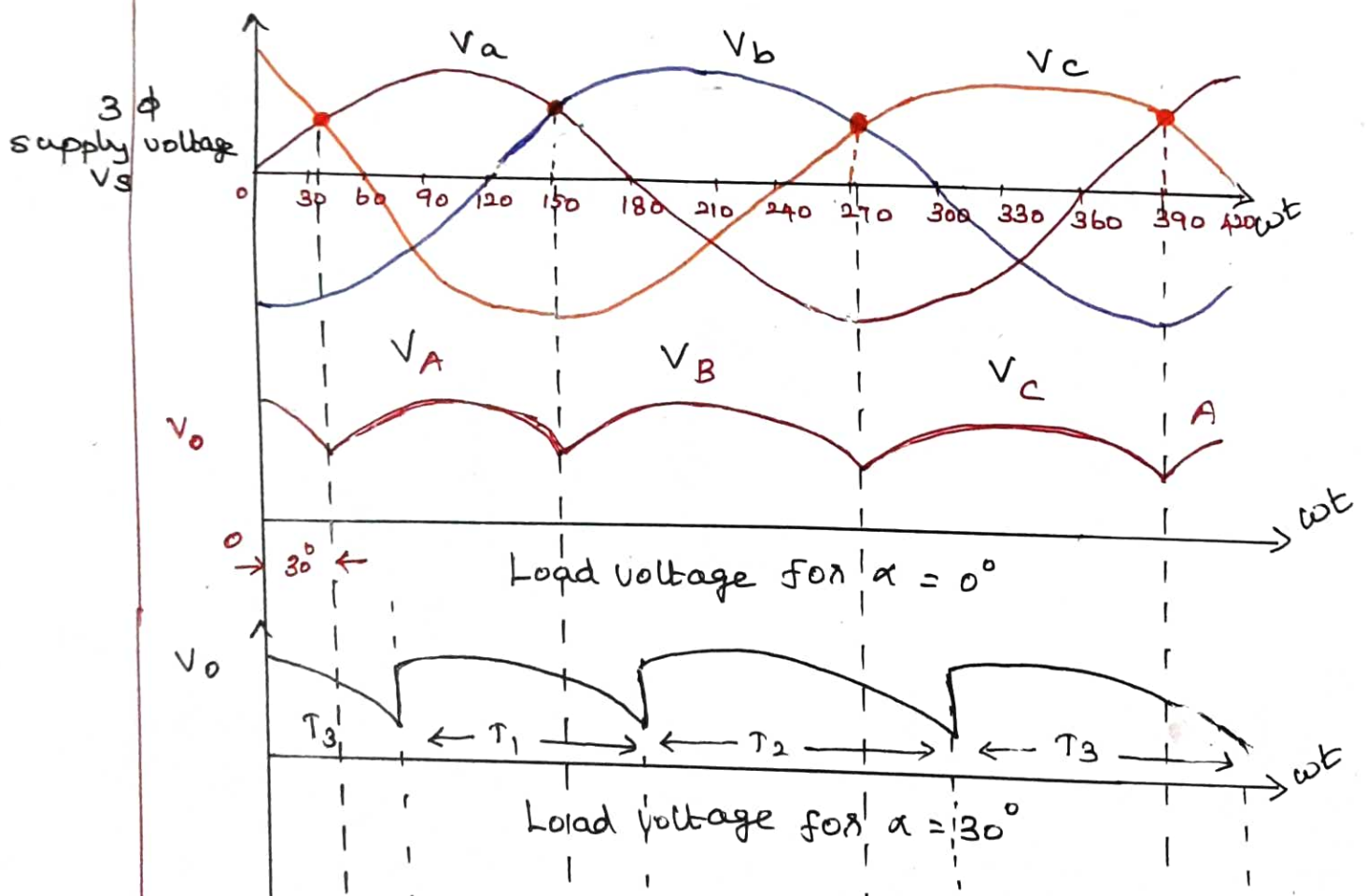
phase A and phase c are equally positive with respect to the neutral.

SCR T_1 connected to phase A cannot be triggered below an angle of 30° , since it is already reverse-biased by the already conducting SCR T_3 . Minimum firing angle is $\pi/6$.

T_1 conducts from $\omega t = 30^\circ$ to $\omega t = 150^\circ$.

T_2 conducts from $\omega t = 150^\circ$ to $\omega t = 270^\circ$.

T_3 conducts from $\omega t = 270^\circ$ to $\omega t = 390^\circ$.



The 3 ϕ Half wave converter combines three single phase half wave controlled rectifiers.

Thyristor T_1 in series with one of the supply phase windings a-n acts as one half wave ^{supply controlled} rectifier.

second thyristor T_2 in series with supply phase windings b-n acts as second half wave controlled rectifier.

Third thyristor T_3 in series with supply phase windings c-n acts as third half wave controlled rectifier.

when thyristor T_1 is triggered at $\omega t = \pi/6 + \alpha$, V_{an} appears across load, and T_1 conducts.

when thyristor T_2 is triggered at $\omega t = 5\pi/6 + \alpha$, T_1 becomes reverse biased and turns off, V_{bn} appears across load, and T_2 conducts.

when thyristor T_3 is triggered, $\omega t = 3\pi/2 + \alpha$, V_{cn} appears across load, T_3 conducts.

$$\begin{aligned}
 V_o &= 3 \times \frac{1}{2\pi} \int_{\alpha + \pi/6}^{5\pi/6 + \alpha} V_m \sin \omega t \cdot d(\omega t) \\
 &= \frac{V_m \times 3}{2\pi} \left[-\cos \omega t \right]_{\pi/6 + \alpha}^{5\pi/6 + \alpha} \\
 &= \frac{V_m \times 3}{2\pi} \left[-\cos \left(\frac{5\pi}{6} + \alpha \right) + \cos \left(\frac{\pi}{6} + \alpha \right) \right] \\
 &= \frac{3V_m}{2\pi} \left[- \left[\cos \frac{5\pi}{6} \cos \alpha + \sin \frac{5\pi}{6} \sin \alpha \right] \right. \\
 &\quad \left. + \left[\cos \frac{\pi}{6} \cos \alpha + \sin \frac{\pi}{6} \sin \alpha \right] \right].
 \end{aligned}$$

$$= \frac{3V_m}{2\pi} \left[+0.866 \cos \alpha - 0.5 \sin \alpha + 0.866 \cos \alpha + 0.5 \sin \alpha \right]$$

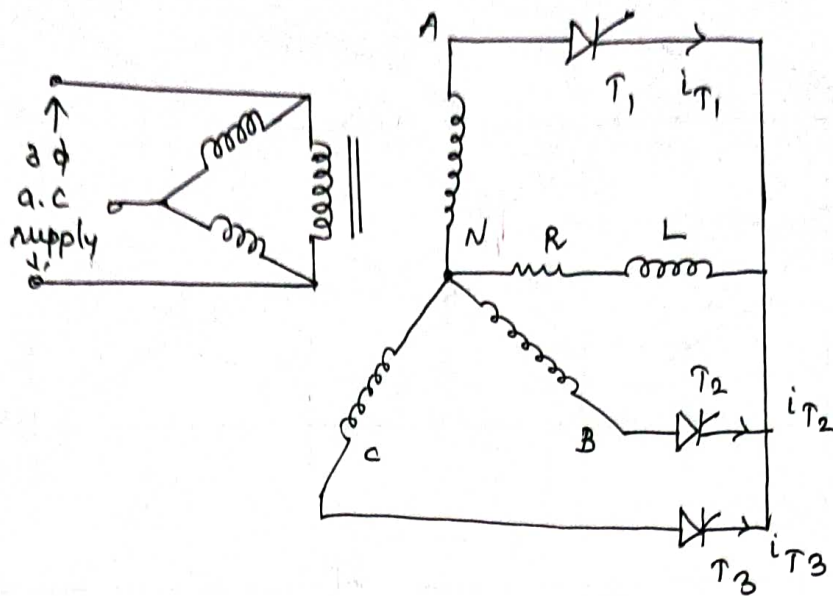
$$= \frac{3V_m}{2\pi} \sqrt{3} \cos \alpha$$

$$V_o = \frac{3\sqrt{3}}{2\pi} V_m \cos \alpha$$

Rms load voltage :

$$\begin{aligned} E_{rms} &= \left[\frac{3}{2\pi} \int_{\alpha+30^\circ}^{\alpha+150^\circ} V_m^2 \sin^2 \omega t \, d(\omega t) \right]^{1/2} \\ &= V_m \sqrt{\frac{3}{2\pi}} \left[\int_{\alpha+30^\circ}^{\alpha+150^\circ} \frac{1 - \cos 2\omega t}{2} \, d\omega t \right]^{1/2} \\ &= V_m \sqrt{\frac{3}{2\pi}} \times \frac{1}{\sqrt{2}} \left[\omega t - \frac{\sin 2\omega t}{2} \right]_{\alpha+30^\circ}^{\alpha+150^\circ} \Bigg|^{1/2} \\ &= \frac{V_m}{2} \sqrt{\frac{3}{\pi}} \left[\alpha+150 - \frac{\sin 2(\alpha+150^\circ)}{2} - [\alpha+30] + \frac{\sin 2(\alpha+30^\circ)}{2} \right]^{1/2} \\ &= \frac{V_m}{2} \sqrt{\frac{3}{\pi}} \left[120^\circ - \frac{\sin 2\alpha \cos 30^\circ}{2} + \frac{\sin 30^\circ \cos 2\alpha}{2} + \frac{\sin 2\alpha \cos 60^\circ}{2} + \frac{\sin 60^\circ \cos 2\alpha}{2} \right]^{1/2} \\ &= \frac{V_m}{2} \sqrt{\frac{3}{\pi}} \left[120^\circ - \frac{0.5 \sin 2\alpha}{2} - \frac{0.866 \cos 2\alpha}{2} + \frac{0.5 \sin 2\alpha}{2} + \frac{0.866 \cos 2\alpha}{2} \right]^{1/2} \\ &= \frac{V_m}{2} \sqrt{\frac{3}{\pi}} \left[\frac{2\pi}{3} + \frac{0.366 \cos 2\alpha}{2} \right]^{1/2} \end{aligned}$$

Three phase Half-wave controlled Rectifier
with inductive load (R-L) :



circuit operation :-

Let the firing angle be say 45° ,

T_1 conducts from $30^\circ + \alpha$ to $150^\circ + \alpha$.

T_2 conducts from $150^\circ + \alpha$ to $270^\circ + \alpha$.

T_3 conducts from $270^\circ + \alpha$ to $390^\circ + \alpha$.

SCR conducts for 120° .

At $\omega t = \pi$, phase voltage V_a is zero,
but i_a is not zero, because of RL load.

Therefore T_1 continue conducting beyond $\omega t = \pi$.

When T_1 is on, $V_{T1} = V_a - V_a = 0$, $\omega t = 75^\circ$ to 195° .

When T_2 is on, $V_{T1} = V_a - V_b$, $\omega t = 195^\circ$ to 315° .

When T_3 is on, $V_{T1} = V_a - V_c$, $\omega t = 315^\circ$ to 435° .

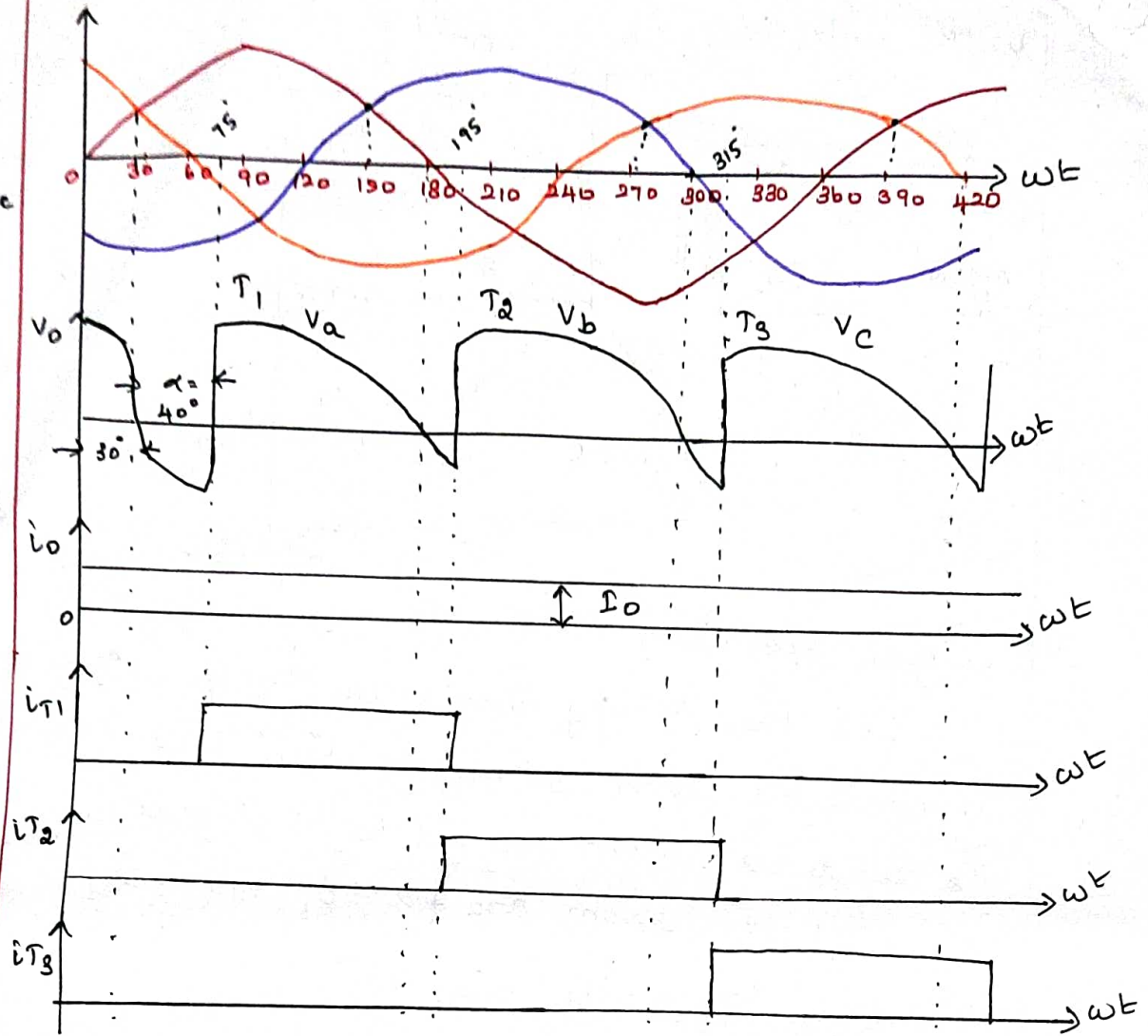
T_2 turned on at $\omega t = 195^\circ$,

$$V_{T1} = V_a - V_b = V_{m\phi} \sin 195^\circ - V_{m\phi} \sin 75^\circ$$

$$= -0.25 V_m - 0.96 V_m$$

$$= -1.215 V_m$$

3 ϕ
supply
voltage
 V_s



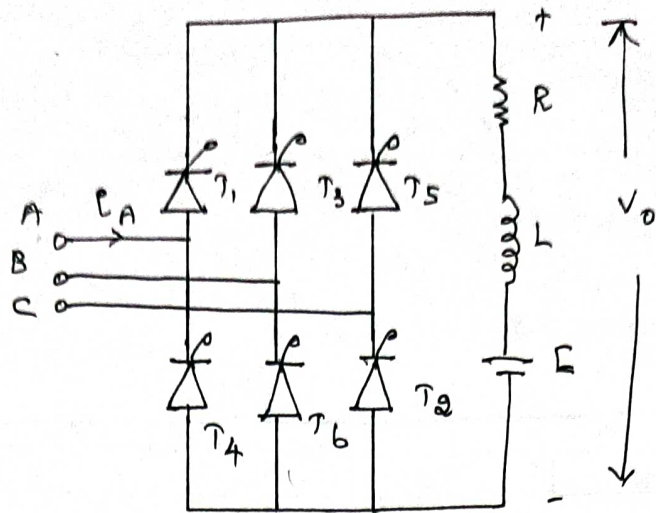
$$5\pi/6 + \alpha$$

$$V_o \text{ (or) } V_{dc} = \frac{3}{2\pi} \int_{\pi/6 + \alpha}^{5\pi/6 + \alpha} v_m \sin \omega t \cdot d(\omega t)$$

$$V_o = \frac{3\sqrt{3}}{2\pi} v_m \cos \alpha$$

$$V_{rms} = \left[\frac{3}{2\pi} \int_{\pi/6 + \alpha}^{5\pi/6 + \alpha} v_m^2 \sin^2 \omega t \cdot d(\omega t) \right]^{1/2}$$

Three Phase Full converters :- (6 pulse converter).



3 ϕ ac to dc converter for firing angle delay
 $0^\circ < \alpha \leq 90^\circ$.

3 ϕ line commutated inverter for $90^\circ < \alpha < 180^\circ$.

1, 3, 5 \rightarrow Positive group of thyristor.

2, 4, 6 \rightarrow Negative group of thyristor.

for $\alpha = 0^\circ$,

T_1 is triggered at $\omega t = 30^\circ$.

T_2 is triggered at $\omega t = 90^\circ$.

T_3 is triggered at $\omega t = 150^\circ$ and so on.

for $\alpha = 60^\circ$;

T_1 is triggered at $\omega t = 30^\circ + 60^\circ = 90^\circ$.

T_2 is triggered at $\omega t = 90^\circ + 60^\circ = 150^\circ$.

T_3 is triggered at $\omega t = 150^\circ + 60^\circ = 210^\circ$

and so on.

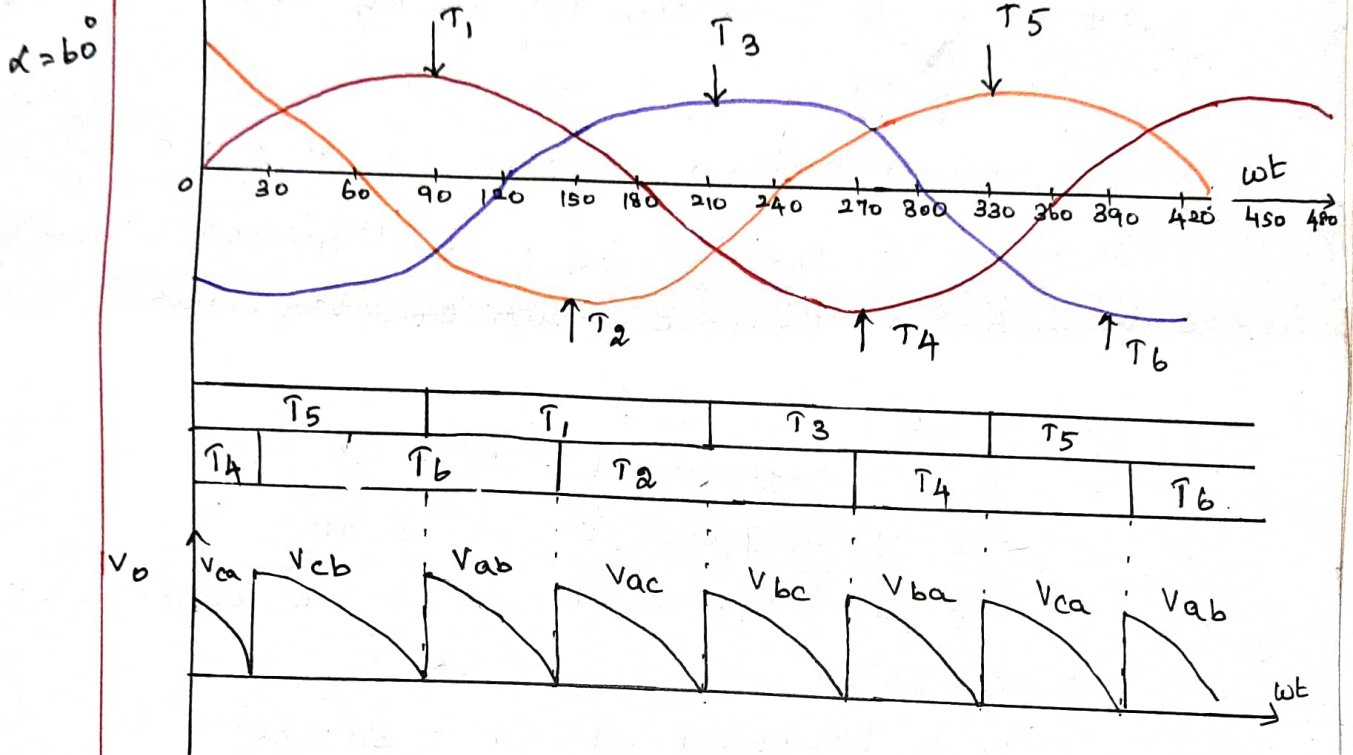
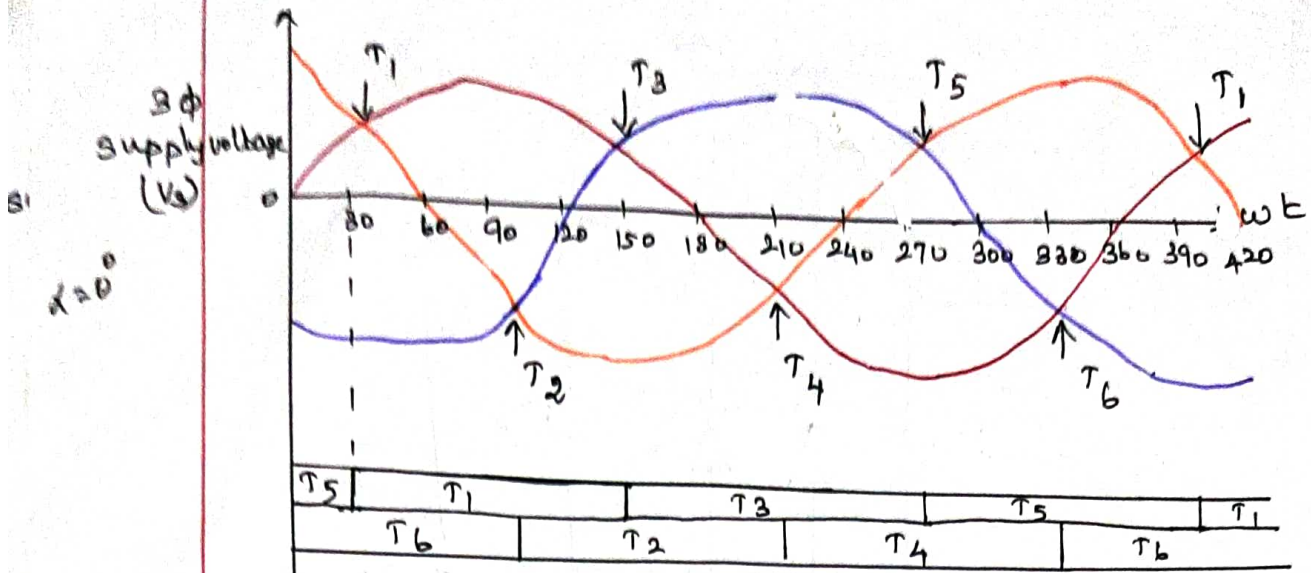
Each SCR is conducts for 180° .

+ve group of SCRs are fired at an interval of 120° .

||| -ve group of SCRs are fired at interval of 120° .

SCR from both the groups are fired at interval of 60°

commutation occurs every 60° .



line to neutral voltages are

$$V_{an} = V_m \sin \omega t$$

$$V_{bn} = V_m \sin \left(\omega t - \frac{2\pi}{3} \right)$$

$$V_{cn} = V_m \sin \left(\omega t + \frac{2\pi}{3} \right)$$

line to line voltages are

$$V_{ab} = V_{an} - V_{bn} = \sqrt{3} V_m \sin(\omega t + \pi/6)$$

$$V_{bc} = V_{bn} - V_{cn} = \sqrt{3} V_m \sin(\omega t - \pi/6)$$

$$V_{ca} = V_{cn} - V_{an} = \sqrt{3} V_m \sin(\omega t + \pi/2)$$

average output voltage is found from,

$$V_o = \frac{3}{\pi} \int_{\pi/2 + \alpha}^{\pi/6 + \alpha} V_{ab} d(\omega t)$$

$$= \frac{3}{\pi} \int_{\pi/6 + \alpha}^{\pi/2 + \alpha} \sqrt{3} V_m \sin(\omega t + \pi/6) d(\omega t)$$

$$= \frac{\sqrt{3} \cdot 3 V_m}{\pi} \int_{\pi/6 + \alpha}^{\pi/2 + \alpha} \sin(\omega t + \pi/6) d(\omega t)$$

$$\frac{\pi}{6} + \frac{\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$$

$$\frac{\pi}{2} + \frac{\pi}{6} = \frac{3\pi}{6} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

$$= \frac{\sqrt{3} \cdot 3 V_m}{\pi} \int_{\pi/3 + \alpha}^{2\pi/3 + \alpha} \sin(\omega t) d(\omega t)$$

$$= \frac{\sqrt{3} \cdot 3 V_m}{\pi} [-\cos \omega t]_{\pi/3 + \alpha}^{2\pi/3 + \alpha}$$

$$= \frac{\sqrt{3} \cdot 3 V_m}{\pi} [\cos(\pi/3 + \alpha) - \cos(2\pi/3 + \alpha)]$$

$$= \frac{\sqrt{3} \cdot 3 \cdot V_m}{\pi} (\cos \pi/3 \cos \alpha - \sin \pi/3 \sin \alpha) -$$

$$\cos 2\pi/3 \cos \alpha - \sin 2\pi/3 \sin \alpha$$

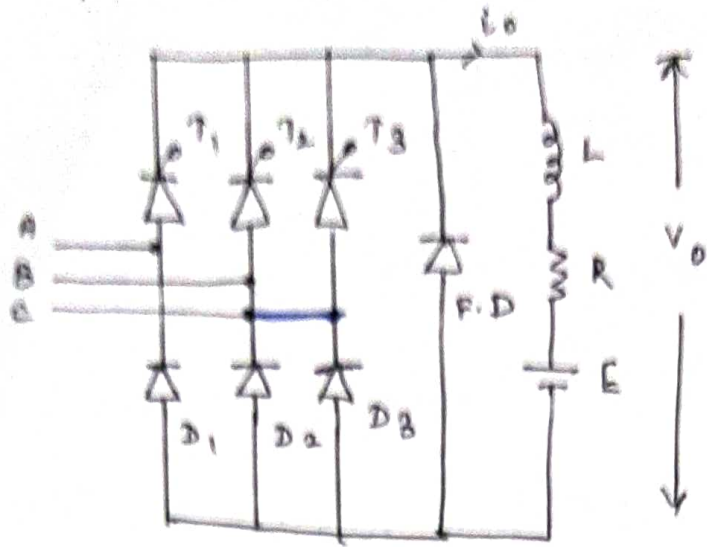
$$= (0.5 \cos \alpha - 0.866 \sin \alpha) + 0.5 \cos \alpha + 0.866 \sin \alpha$$

$$V_o = \frac{\sqrt{3} \cdot 3 V_m}{\pi} \cos \alpha$$

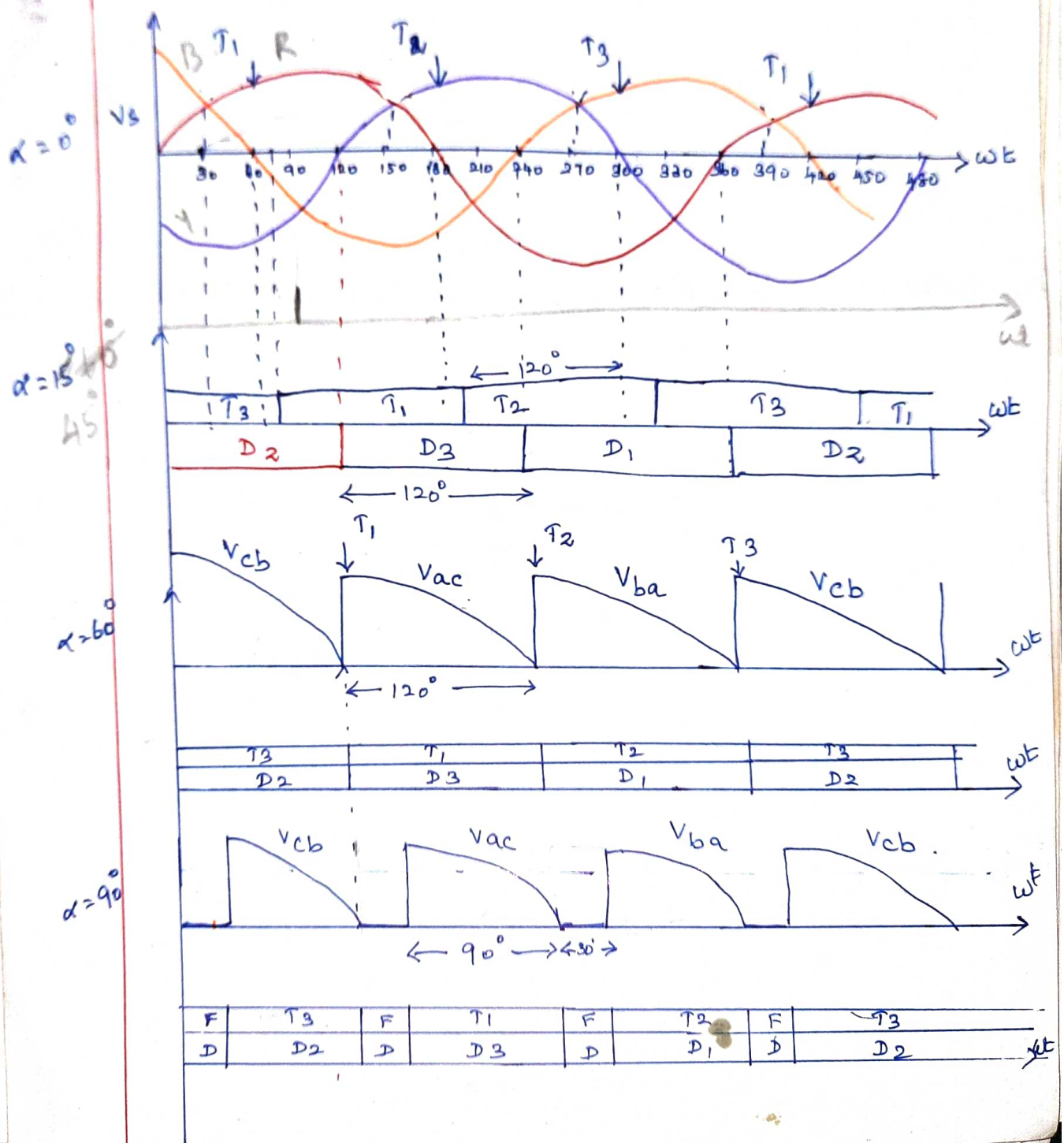
$$\frac{\pi}{2} + \frac{\pi}{6}$$

$$\frac{3\pi}{6} + \frac{\pi}{6}$$

Three phase semiconverter :



$$\frac{72}{6 \times 30} = \frac{7 \times 100}{210}$$



3 ϕ semiconverter are used in industrial applications up to the 120 kW level. The delay angle α can be varied from 0 to π . During the period $\frac{\pi}{6} \leq \omega t < \frac{7\pi}{6}$ T_1 is forward biased.

T_1 is fired at $\omega t = (\frac{\pi}{6} + \alpha)$

T_1, D_3 conducts. V_{ac} appears across load.

At $\omega t = \frac{7\pi}{6}$, V_{ac} starts to negative. Free-wheeling diode D_m conducts.

3 line-neutral voltages as follows :

$$V_{an} = V_m \sin \omega t$$

$$V_{bn} = V_m \sin \left(\omega t - \frac{2\pi}{3} \right)$$

$$V_{cn} = V_m \sin \left(\omega t + \frac{2\pi}{3} \right)$$

$$V_{ac} = V_{an} - V_{cn}$$

$$= \sqrt{3} V_m \sin \left(\omega t - \frac{\pi}{6} \right)$$

$$V_{dc} = \frac{3}{2\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{7\pi}{6}} V_{ac} \cdot d(\omega t) \dots$$

$$= \frac{3}{2\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{7\pi}{6}} \sqrt{3} \cdot V_m \sin \left(\omega t - \frac{\pi}{6} \right) \cdot d(\omega t)$$

$$= \frac{3\sqrt{3}}{2\pi} V_m \int_{\frac{\pi}{6} + \alpha}^{\frac{7\pi}{6}} \sin \left(\omega t - \frac{\pi}{6} \right) \cdot d(\omega t)$$

$$= \frac{3\sqrt{3} V_m}{2\pi} (1 + \cos \alpha)$$

Three-phase semiconverter :-

Performance Parameters :-

1) Input Displacement Angle (ϕ_1):

The angular displacement between the fundamental component of the a.c line current and the associated line to neutral voltage.

2) Input Displacement Factor ($\cos \phi_1$):

The input displacement factor is defined as the cosine of the input displacement angle.

3) Input power factor :

It is defined as the ratio of the total mean input power to the total RMS input volt-ampere.

$$P.F = \frac{E_1 I_1 \cos \phi_1}{E_{rms} I_{rms}}$$

4) DC Voltage Ratio (r):

It is defined as the ratio of mean d.c terminal voltage at a given firing angle α to the maximum possible d.c terminal voltage.

5) Input current Distortion factor :

It is defined as ratio of the RMS amplitude of the fundamental component to the total RMS amplitude.

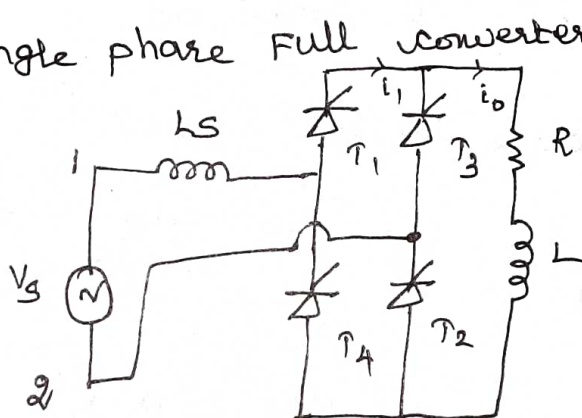
Effect of source impedance on the performance of converter :-

Incoming SCRs T_1 and T_2 are fired in a 1 ϕ full converter, outgoing SCRs T_3 and T_4 get turned off. This is possible only if the voltage source has no internal impedance.

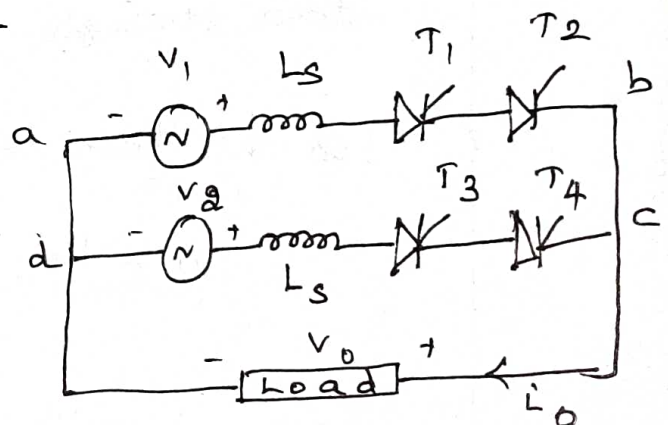
If the source impedance is resistive, there will be a voltage drop across the resistance, average voltage output of a converter gets reduced by an amount equal to $I_o r_s$, for 1 ϕ converter, and by $2 I_o r_s$ for 3 ϕ converter.

If the source impedance is inductive, it causes the outgoing and incoming SCR to conduct together. The commutation period in seconds, when outgoing and incoming SCRs are conducting together, is also known as commutation angle or overlap angle (μ) in degrees or radians.

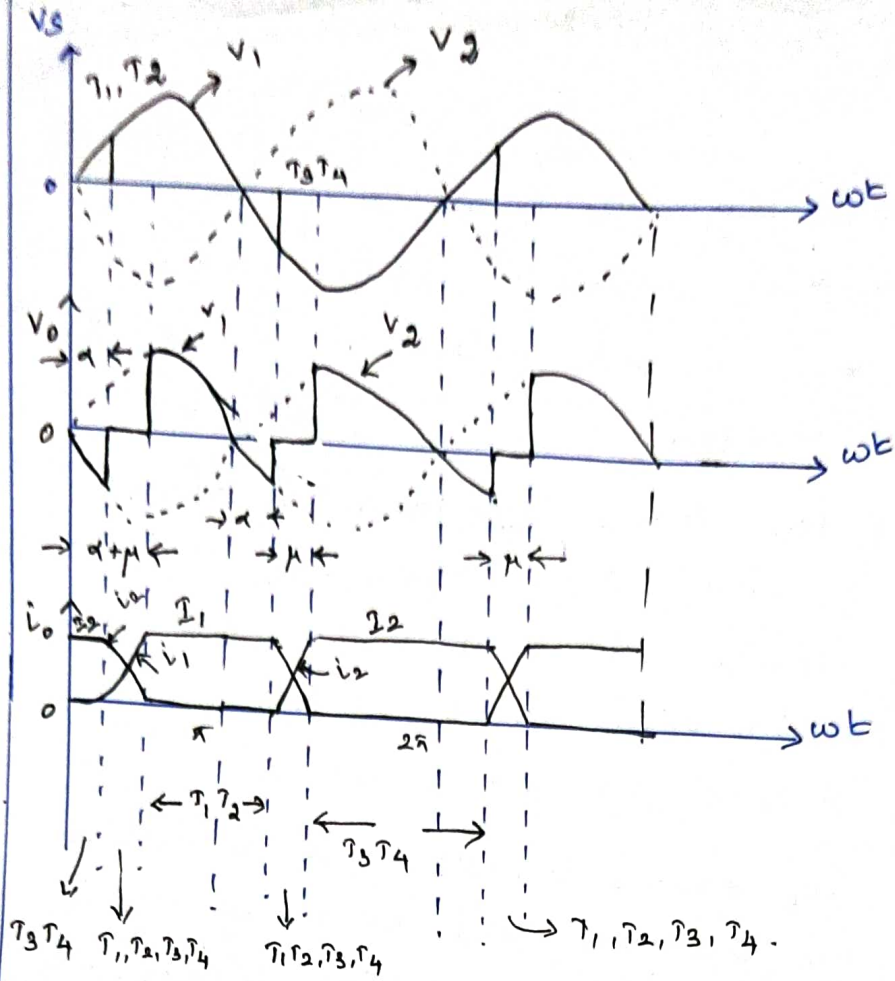
single phase Full converter :-



1 ϕ full converter with source inductance (L_s)



equivalent circuit



when terminal 1 of source voltage (V_s) is positive, current i_1 flows through L_s, T_1, T_2 and load.

when terminal 2 of V_s is positive, load current i_2 flows through L_s, T_3, T_4 , and load.

for the loop abcda gives

$$V_1 - L_s \cdot \frac{di_1}{dt} = V_2 - L_s \cdot \frac{di_2}{dt}$$

$$V_1 - V_2 = L_s \left[\frac{di_1}{dt} - \frac{di_2}{dt} \right]$$

$$V_1 = V_m \sin \omega t$$

$$V_2 = -V_m \sin \omega t$$

$$L_s \left[\frac{di_1}{dt} - \frac{di_2}{dt} \right] = 2 V_m \sin \omega t$$

Load current is assumed constant,

$$i_1 + i_2 = I_0$$

diff. w.r. to t , we get,

$$\frac{di_1}{dt} + \frac{di_2}{dt} = 0 \dots \dots (1)$$

$$\frac{di_1}{dt} - \frac{di_2}{dt} = \frac{2V_m}{L_s} \sin \omega t \dots \dots (2)$$

$$\textcircled{1} + \textcircled{2} \Rightarrow \frac{di_1}{dt} = \frac{V_m}{L_s} \sin \omega t \dots \dots (3)$$

Load current i_1 through thyristor pair T_1, T_2 builds up from zero to I_0

at $\omega t = \alpha, i_1 = 0$

at $\omega t = \alpha + \mu, i_1 = I_0$

$$[I_0 = I_1]$$

From eqn ③ $\int_0^{I_0} \frac{di_1}{dt} \cdot dt = \int_{\alpha/\omega}^{\alpha+\mu/\omega} \frac{V_m}{L_s} \sin \omega t \cdot dt$

$$(\cos \alpha - \cos(\alpha + \mu)) \cdot \frac{I_0 \omega L_s}{I_0} = \frac{V_m}{L_s} [-\cos \omega t]_{\alpha/\omega}^{\alpha+\mu/\omega}$$

$$-\cos(\alpha + \mu) = \frac{I_0 \omega L_s}{I_0} - \cos \alpha \quad I_0 = \frac{V_m}{\omega L_s} [\cos \alpha - \cos(\alpha + \mu)] \dots \dots (3a)$$

$$\cos(\alpha + \mu) = \cos \alpha - \frac{I_0 \omega L_s}{I_0}$$

From the figure, V_o is zero from $\alpha + \pi$ to $\alpha + \mu$.

$$V_{oav} = \frac{V_m}{\pi} \int_{\alpha+\mu}^{\alpha+\pi} \sin \omega t \cdot d(\omega t)$$

$$= \frac{V_m}{\pi} [\cos(\alpha + \mu) - \cos(\alpha + \pi)]$$

$$= \frac{V_m}{\pi} [\cos \alpha + \cos(\alpha + \mu)] \dots \dots (4)$$

Average value of ^{output} ~~current~~ voltage at no load,

$$V_o = \frac{2V_m}{\pi} \cos \alpha.$$

Maximum mean output voltage, $V_{om} = \frac{2V_m}{\pi}$

$$V_{ox} = \frac{V_m}{2}$$

$$V_m = \frac{V_{om} \times \pi}{2}$$

from eqn (4),

$$V_{ox} = \frac{\text{Maximum mean o/p voltage at no load}}{2}$$

$$\cos \alpha + \cos(\alpha + \mu).$$

$$V_{ox} = \frac{V_{om}}{2} [\cos \alpha + \cos(\alpha + \mu)] \dots (5)$$

from eqn 3A,

$$\cos(\alpha + \mu) = \cos \alpha - \frac{\omega L_s}{V_m} I_o \dots (6)$$

sub (6) in (5),

$$V_{ox} = \frac{2V_m}{\pi} [\cos \alpha + 1]$$

$$V_{ox} = \frac{V_{om}}{2} \left[\cos \alpha + \cos \alpha - \frac{\omega L_s}{V_m} I_o \right]$$

$$= \frac{2V_{om}}{2\pi} \cos \alpha - \frac{\omega L_s}{V_m} I_o$$

$$V_{ox} = \frac{2V_m}{\pi} \cos \alpha - \frac{\omega L_s}{V}$$

$$V_{ox} = \frac{V_m}{\pi} \left[\cos \alpha + \cos \alpha - \frac{\omega L_s}{V_m} I_o \right]$$

$$= \frac{2V_m}{\pi} \cos \alpha - \frac{\omega L_s}{\pi V_m} I_o$$

Voltage regulation due to source inductance

$$= \frac{\omega L_s}{\pi} \times I_0 \times \frac{1}{V_0 \text{ at no load}}$$

$$= \frac{2\pi f L_s I_0}{\pi} \times \frac{\pi}{2V_m \cos \alpha}$$

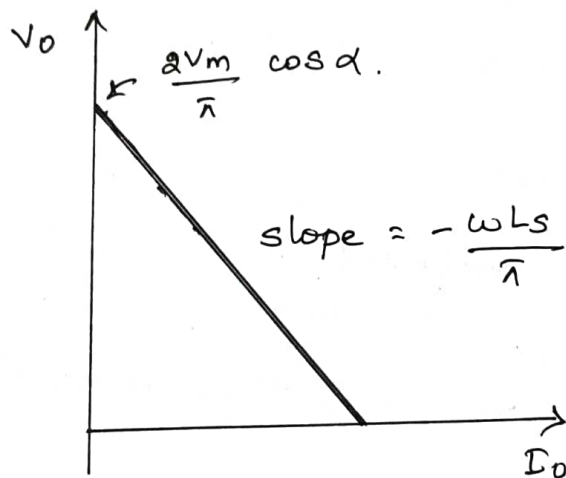
$$= \frac{\pi f L_s I_0}{V_m \cos \alpha}$$

From eqn (3a)

$$\frac{\omega L_s I_0}{\pi} = \frac{V_m}{\pi} [\cos \alpha - \cos(\alpha + \mu)]$$

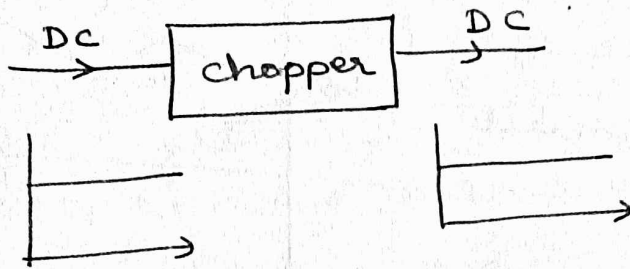
For full wave diode rectifier, $\alpha = 0^\circ$,

$$\frac{\omega L_s I_0}{\pi} = \frac{V_m}{\pi} [1 - \cos \mu]$$



UNIT - III Choppers

The conversion of fixed dc voltage to an adjustable dc output voltage through the use of semiconductor devices can be carried out by the use of dc-dc converters.



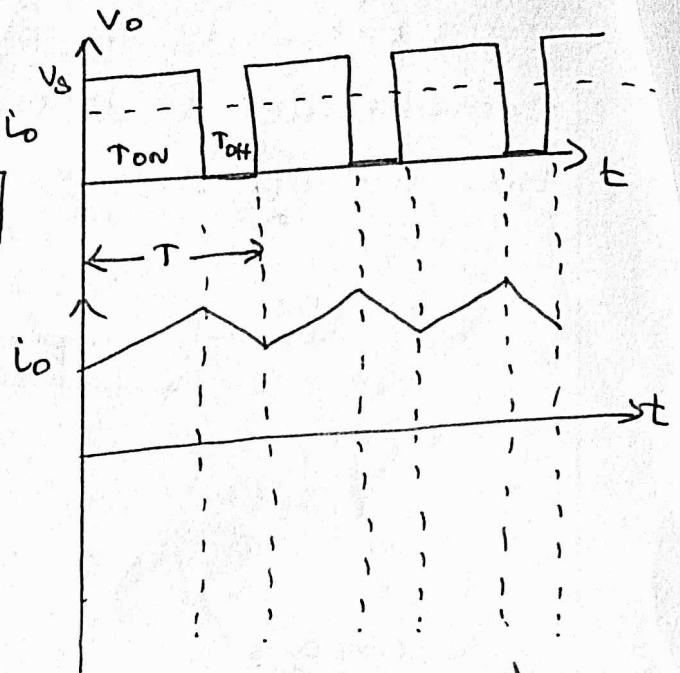
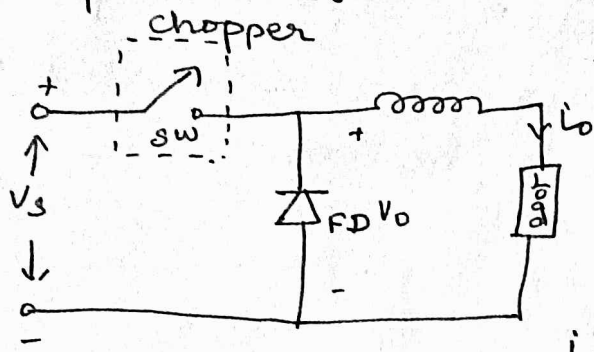
DC chopper :-

A chopper is a static device that converts fixed dc input voltage to a variable dc output voltage directly.

- dc equivalent of an ac transformer.
- The power semiconductor devices used for a chopper circuit can be power BJT, power MOSFET, GTO or force commutated thyristor.
- These devices in general, can be represented by a switch SW with an arrow.
- When the switch is off, no current can flow.
- When the switch is on, current flows in the direction of arrow only.
- ON state voltage drop 0.5V to 2.5V.

Step-Down Chopper

The average output voltage is less than d.c input voltage is known as step down chopper



Output voltage and current waveforms.

⇒ During the period T_{on} , chopper is on and load voltage is equal to source voltage V_s .

⇒ During the interval T_{off} , chopper is off, load current flows through the freewheeling diode FD.

⇒ So, load terminals are short circuited by FD and the load voltage is zero during T_{off} .

The average load voltage

$$V_o = \frac{T_{on}}{T_{on} + T_{off}} V_s = \frac{T_{on}}{T} V_s$$

$$= d V_s.$$

T_{on} → on time, T_{off} → off time.

$T = T_{on} + T_{off}$ = Chopping period.

$$\alpha = \frac{T_{on}}{T} = \text{duty cycle}$$

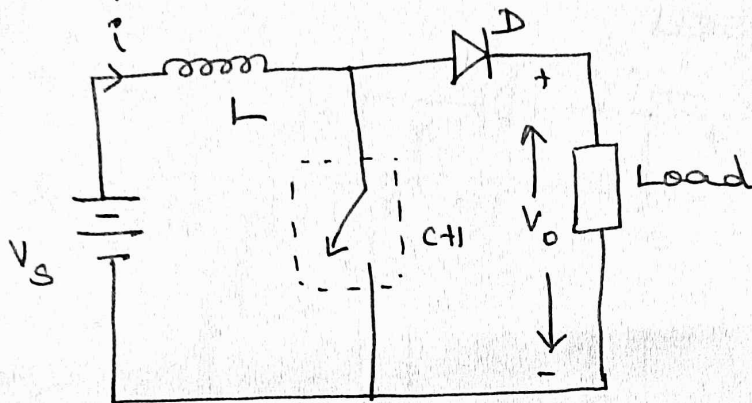
\Rightarrow Load voltage can be controlled by varying duty cycle α .

$$\therefore V_o = f \cdot T_{on} \cdot V_s$$

$$T = \frac{1}{f} = \text{chopping frequency}$$

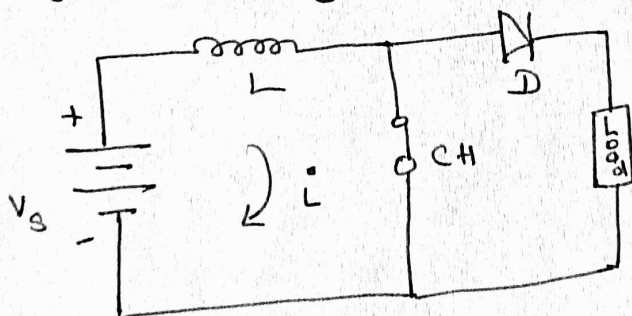
Step up choppers :-

The average output voltage V_o is ^{greater} less than the input voltage. $V_o > V_s$.



step-up chopper.

When the chopper CH is on, the inductor stores energy during T_{on} period.



When CH is on, current through the load increase from I_1 to I_2 .

$$V_L = V_s$$

The energy input to inductor from the source during the period T_{on} , is

$$W_{in} = (\text{Voltage across } L) (\text{average current through } L) \times T_{on}.$$

$$= V_s \times \left[\frac{I_1 + I_2}{2} \right] \times T_{on}.$$

When the ch is off, the inductor current cannot die down instantaneously. As the current tends to decrease, polarity of the emf induced in L is reversed.

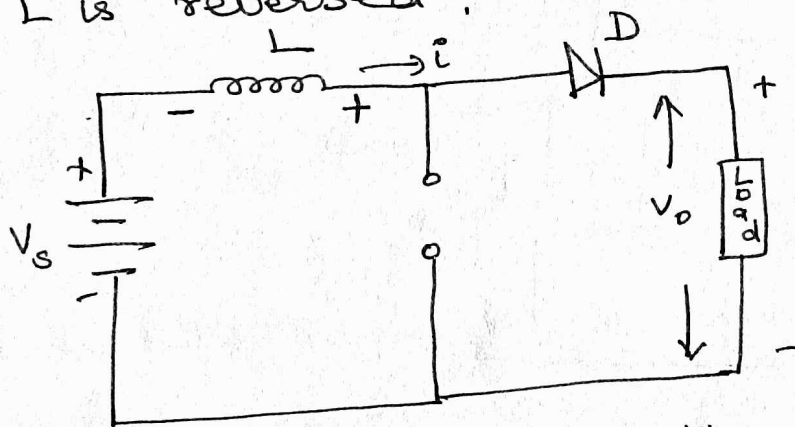


fig: $L \cdot \frac{di}{dt}$ is added to V_s .

voltage across the load is,

$$V_o = V_s + L \cdot \frac{di}{dt} \dots > \text{source voltage } V_s.$$

KCL for the circuit is,

$$V_L - V_o + V_s = 0.$$

$$\boxed{V_L = V_o - V_s}$$

The energy released by inductor to the load is

$$W_{off} = (\text{Voltage across } L) (\text{average current through } L) T_{off}.$$

$$= (V_o - V_s) \left[\frac{I_1 + I_2}{2} \right] T_{\text{off}}$$

Considering the system to be lossless,

$$V_s \left[\frac{I_1 + I_2}{2} \right] T_{\text{on}} = (V_o - V_s) \left[\frac{I_1 + I_2}{2} \right] T_{\text{off}}$$

$$\left[\frac{V_s I_1}{2} + \frac{V_s I_2}{2} \right] T_{\text{on}} = \left[\frac{V_o I_1}{2} + \frac{V_o I_2}{2} - \frac{V_s I_1}{2} - \frac{V_s I_2}{2} \right] T_{\text{off}}$$

$$\frac{V_s I_1}{2} T_{\text{on}} + \frac{V_s I_2}{2} T_{\text{on}} = \frac{V_o I_1}{2} T_{\text{off}} + \frac{V_o I_2}{2} T_{\text{off}} - \frac{V_s I_1}{2} T_{\text{off}} - \frac{V_s I_2}{2} T_{\text{off}}$$

$$\frac{V_s I_1}{2} [T_{\text{on}} + T_{\text{off}}] + \frac{V_s I_2}{2} [T_{\text{on}} + T_{\text{off}}] = \frac{V_o I_1}{2} T_{\text{off}} + \frac{V_o I_2}{2} T_{\text{off}}$$

$T_{\text{on}} + T_{\text{off}} = T$
 $T_{\text{off}} = T - T_{\text{on}}$

$$\frac{V_s I_1}{2} T + \frac{V_s I_2}{2} T = [T - T_{\text{on}}] \left[\frac{V_o I_1}{2} + \frac{V_o I_2}{2} \right]$$

$$T \left[\frac{V_s I_1}{2} + \frac{V_s I_2}{2} \right] = [T - T_{\text{on}}] \left[\frac{V_o I_1}{2} + \frac{V_o I_2}{2} \right]$$

$$\frac{T}{T - T_{\text{on}}} = \frac{\left[\frac{V_o I_1}{2} + \frac{V_o I_2}{2} \right]}{\left[\frac{V_s I_1}{2} + \frac{V_s I_2}{2} \right]}$$

$$= \frac{V_o I_1 + V_o I_2}{V_s I_1 + V_s I_2} = \frac{V_o (I_1 + I_2)}{V_s (I_1 + I_2)}$$

$$V_o = V_s \times \frac{T}{T - T_{\text{on}}}$$

$$\alpha = \frac{T_{ON}}{T} \Rightarrow T_{ON} = \alpha T$$

$$V_o = \frac{V_s T}{T - T_{ON}} = V_s \times \frac{T}{T - \alpha T}$$

$$= V_s \times \frac{T}{T} \left[\frac{1}{1 - \alpha} \right]$$

$$V_o = V_s \times \frac{1}{1 - \alpha}$$

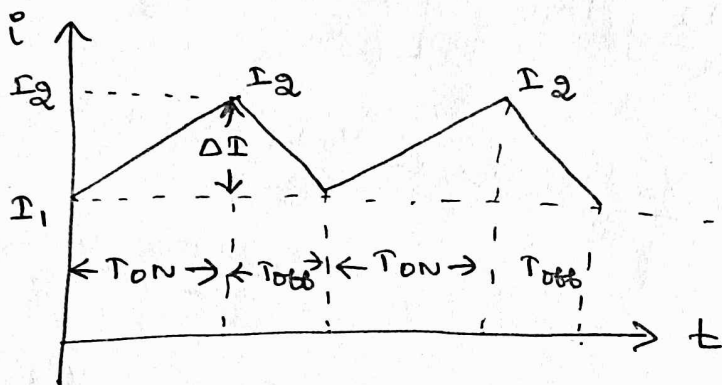


Fig: current waveform.

Step Down Chopper

Express the following variables as function of V_s , R , duty cycle α in case of load is resistive:-

a) Average output voltage $V_o = \frac{T_{ON}}{T} V_s = \alpha V_s$.

b) Average output current $I_o = \frac{V_o}{R} = \frac{T_{ON}}{T} \cdot \frac{V_s}{R}$

$$= \frac{\alpha V_s}{R}$$

c) output current at the instant of commutation
 $i_s = V_s / R$.

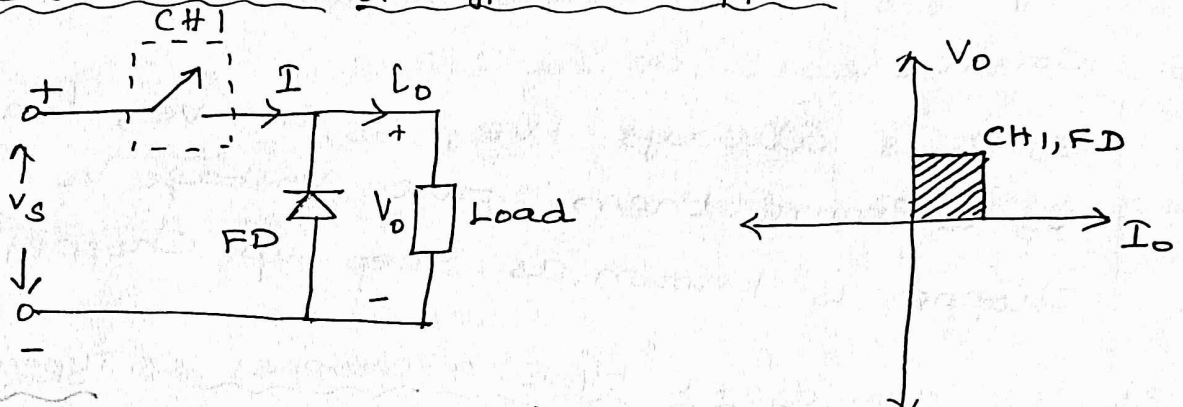
d) For a resistive load, average & rms value of freewheeling diode currents are zero.

Types of Chopper Circuits.

The various chopper configurations are

- 1) First Quadrant or Type-A Chopper
- 2) Second Quadrant or Type-B Chopper
- 3) Two Quadrant type A chopper or Type C chopper.
- 4) Two Quadrant type B Chopper or Type D Chopper.
- 5) Four quadrant chopper or Type E Chopper.

1) First Quadrant or Type A Chopper :-



First Quadrant or type A chopper

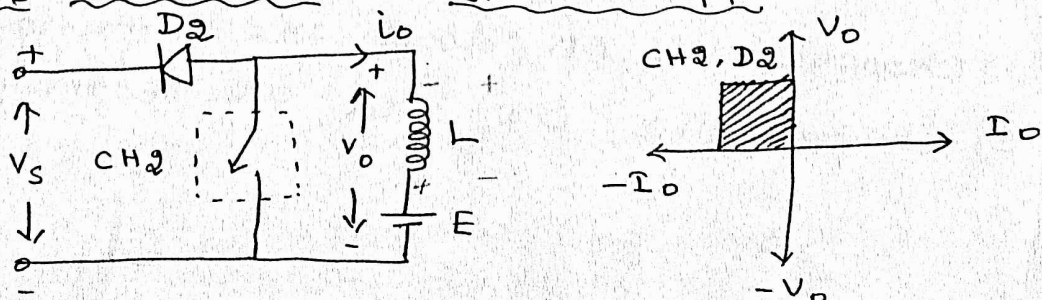
When chopper CH_1 is on, $V_o = V_s$. Current i_o flows in the arrow direction.

When CH_1 is off, $V_o = 0$, but i_o continues flowing in the same direction through freewheeling diode.

Power flow is always from source to load.

V_o , i_o are always positive. This chopper is also called step down chopper.

2) Second Quadrant or Type B Chopper :-



Load must contain a d.c source E , (battery or a d.c motor) in this chopper.

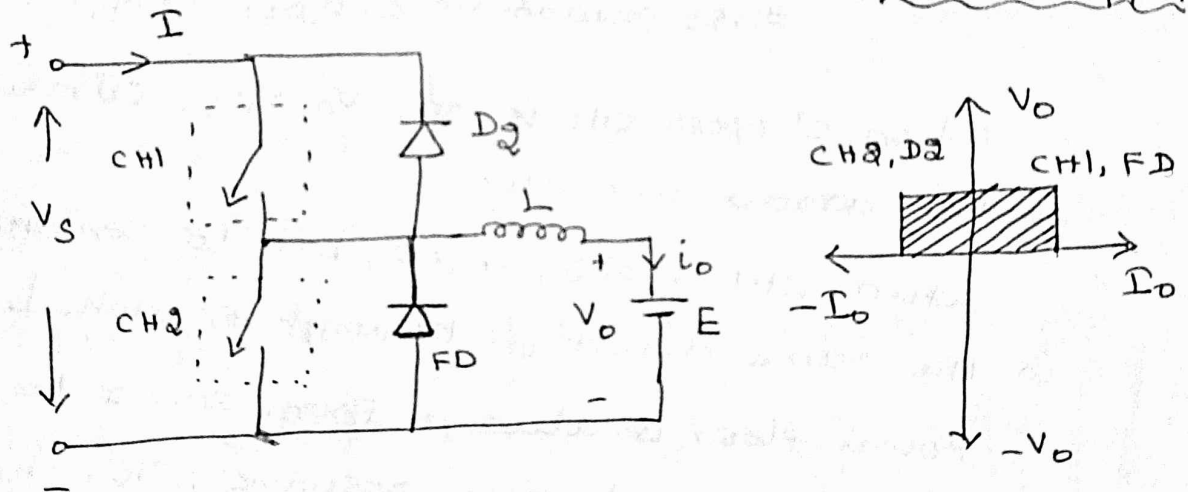
\Rightarrow when CH_2 is ON, $V_o = 0$. Load voltage E drives current through L and CH_2 .

\Rightarrow Inductance L stores energy during T_{on} of CH_2 .

\Rightarrow when CH_2 is off, $V_o = E + L \cdot \frac{di}{dt}$ exceeds source voltage V_s , so Diode D_2 is forward biased and begins conduction. Power flow from load to source.

\Rightarrow V_o is always +ve, I_o is -ve, load voltage V_o is more than source voltage V_s , so this chopper is known as step-up chopper.

3) Two Quadrant type-A chopper or Type C chopper:



\Rightarrow This type of chopper is obtained by connecting type A and type B choppers in parallel.

\Rightarrow Chopper CH_1 , D_1 , load are forming class A chopper.

\Rightarrow Chopper CH_2 , D_2 , load are forming class B chopper.

Case 1: When CH_1 is switched ON/OFF

When CH_1 is switched ON, V_s directly gets connected to load, $V_o = V_s$. The direction of load current is from source to load.

When CH_1 is switched OFF, FD comes into the circuit, it is forward biased, so, $V_o = 0$.

I_o is decreased through FD & L in the same direction. V_o, I_o are positive.

Case 2: When CH_2 is switched ON/OFF

When CH_2 is switched ON, load DC source E drives current through CH_2 and load. i_o direction opposite and is assumed negative.

$V_o = 0$.

When CH_2 is switched OFF, D_2 gets forward biased, current flows from load to source.

$V_o = V_s$. load current is always negative.

So, from the above two cases;

- 1) $V_o = 0$, when CH_2 is ON, (or) FD conducts.
- 2) $V_o = V_s$, when CH_1 is ON (or) Diode D_1 conducts.
- 3) I_o is +ve, when CH_1 is ON or FD conducts.
- 4) I_o is -ve, when CH_2 is ON or D_2 conducts.

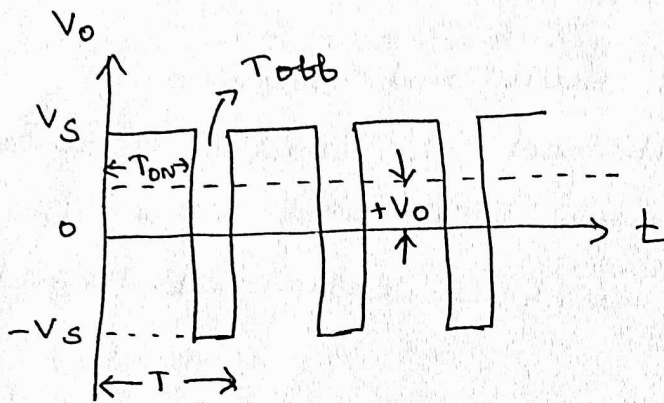
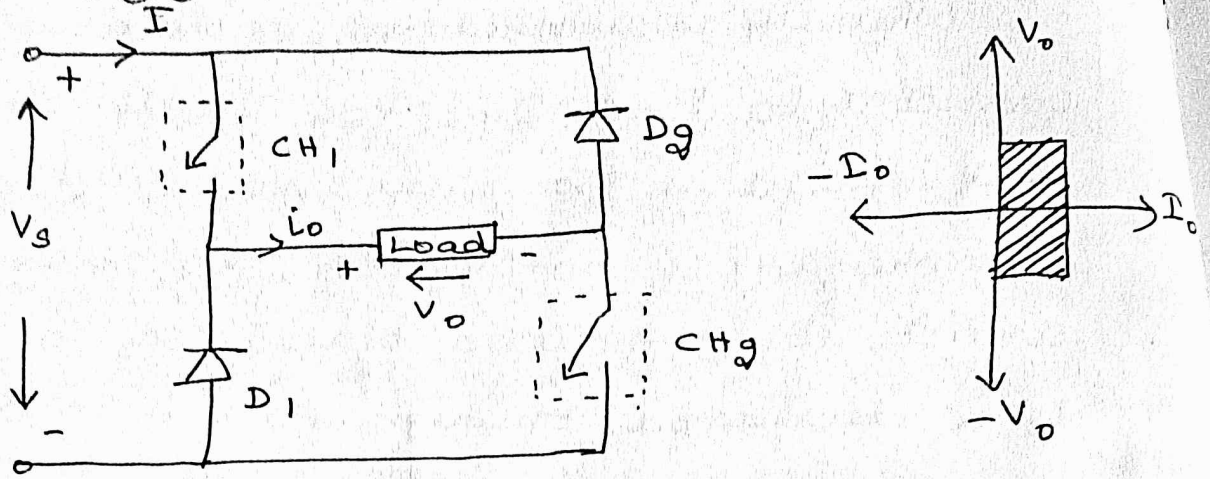
⇒ The average load voltage is always +ve, average load current may be positive or negative.

Caution :-

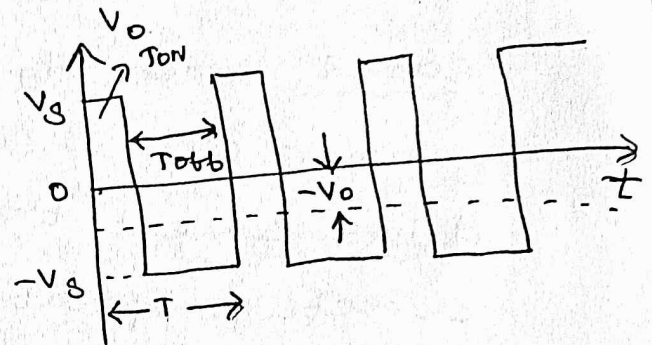
Choppers CH_1 & CH_2 should not be ON simultaneously, this would lead to direct short circuit on the supply lines.

Application :- Used for Motoring and regenerative braking of DC Motors.

A) Two quadrant Type B chopper or Type D chopper



V_o is +ve, $T_{on} > T_{off}$



V_o is -ve, $T_{on} < T_{off}$

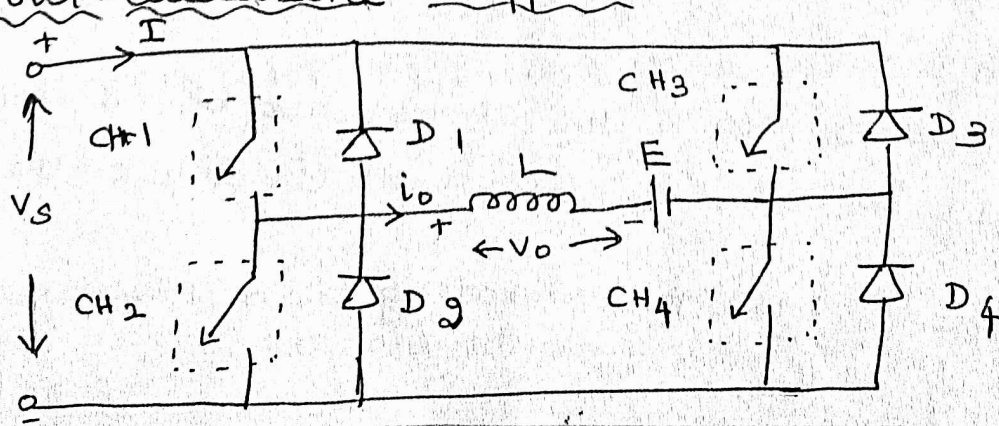
1) The output voltage $V_o = V_s$, when both CH_1 & CH_2 are ON. and $V_o = -V_s$, when both CH_1 & CH_2 are OFF, & both Diodes D_1 , D_2 conduct.

2) V_o is positive, when $T_{on} > T_{off}$

3) V_o is negative, when $T_{on} < T_{off}$

4) Direction of load current is always +ve, V_o is reversible, power flow is reversible. The operation is in first and fourth quadrants.

5) Four-Quadrant chopper or Type E chopper :-



It consists of four semiconductor switches CH_1 to CH_4 and four diodes D_1 to D_4 in antiparallel.

First Quadrant:

CH_4 is kept ON, CH_3 - kept OFF, CH_1 operated.
with CH_1 , CH_4 ON, load voltage $V_o = V_s$. Load current i_o begins to flow. Both V_o & i_o are +ve.
When CH_1 is OFF, +ve current freewheels through CH_4 , D_2 .

Second Quadrant:

CH_2 → operated, CH_1 , CH_3 , CH_4 are kept OFF.
with CH_2 ON reverse current flows through L , CH_2 , D_4 & E . Inductance L stores energy during the time CH_2 is ON.

CH_2 → OFF, current fed back to source through diodes D_1 , D_4 . Here $(E + L \cdot \frac{di}{dt})$ is more than the source voltage V_s . As load voltage V_o is +ve & I_o is -ve.

Third Quadrant:-

CH_1 → OFF, CH_2 → ON, CH_3 → operated.
Polarity of load emf E must be reversed. with CH_3 ON, load gets connected to source V_s . so both V_o , i_o are negative.

CH_3 → OFF, negative current freewheels through CH_2 , D_4 .

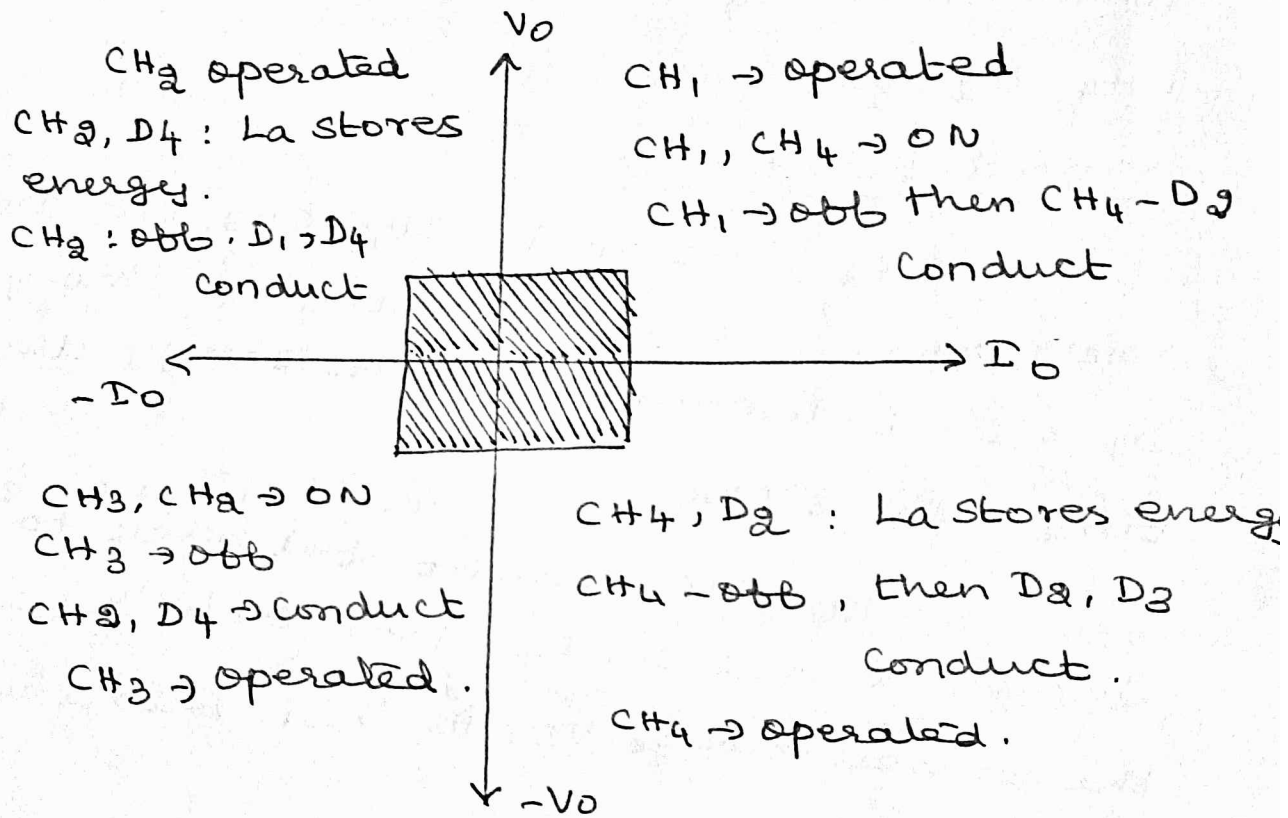
Fourth Quadrant:-

CH_4 is operated and other devices are kept OFF. with CH_4 ON, +ve current flows through

CH_4, D_2, L and E . Inductance L stores energy during the time CH_4 is ON. E

$CH_4 \rightarrow off$, current is fed back to source through diodes D_2, D_3 .

Load voltage is negative, load current is Power is fed back from load to source.



operation of conducting devices.

RESONANT CONVERTER

Generally in converters, the power devices are made to turn ON and turn OFF the entire load current with high di/dt or with high dv/dt . This increases the power losses in the switching device.

In order to minimize this effects, the power devices are turned ON and OFF, when the voltage across it or current through it is zero at the instant of switching. The converter circuits which employs zero-voltage and zero-current switching are called resonant converters.

The resonant converters are of two types:

They are

- (i) zero current switching (ZCS)
- (ii) zero voltage switching (ZVS).

Zero current switching resonant converters:

There are two types of ZCS resonant converters.

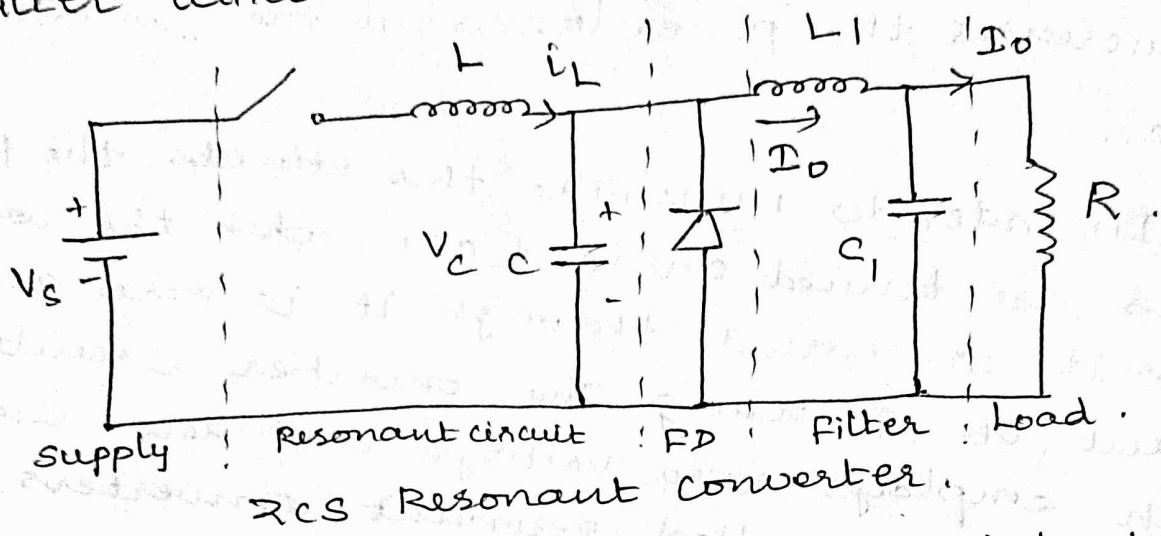
They are

- 1) L-type
- 2) M-type.

L type ZCS Resonant Converters :-

The L-type ZCS Resonant converter is shown in the figure. The switching device S can be any one of the power semiconductor

devices like GTO, Thyristor, BJT, MOSFET. Inductor L and capacitor C near the d.c source (V_s) form a resonant circuit whereas L_1, C_1 near the load constitute a filter circuit.

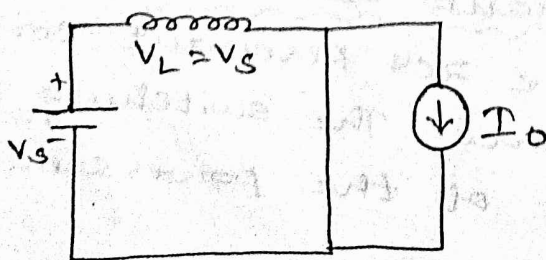


The current in the output inductor L_1 is assumed to be ripple free and equal to the load current I_o .

Initially when the switch is open, the diode is forward biased, to carry the output inductor current and the voltage across C is zero.

Analysis for $0 \leq t \leq t_1$

When the switch is ON, the diode initially remains forward-biased to carry I_o and the voltage across L is the same as that of the source voltage V_s .



The switch is closed at $t=0$, the diode is ON, the voltage across L is V_s . The current in L is expressed as,

$$i_L(t) = \frac{1}{L} \int V_s \cdot dt$$

$$i_L(t) = \frac{V_s}{L} \int dt$$

$$\boxed{i_L(t) = \frac{V_s}{L} t} \quad \text{--- (1)}$$

at $t=t_1$, i_L reaches I_0 , hence

$$i_L(t) = I_0$$

Sub $i_L(t) = I_0$
 $t = t_1$ in
eqn (1).

$$I_0 = \frac{V_s}{L} t_1$$

$$\boxed{t_1 = \frac{I_0 \cdot L}{V_s}}$$

Analysis for $t_1 \leq t \leq t_2$:-

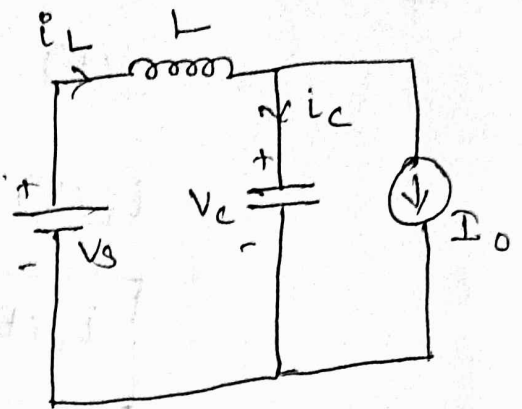
The current in L increases linearly and the diode remains forward biased. when i_L reaches I_0 , the diode turns off.

If I_0 is a constant, the load appears as a current source and the LC circuit oscillates. Consequently i_L returns to zero and remains there. The switch is turned off after the current reaches zero, resulting in zero current switching and no switching power loss.

When the diode (D) current reduces to zero the resonant capacitor C is charged resonant by a current ($i_L - I_0$). The inductor current is given by,

$$\textcircled{1} \leftarrow V_c(t) = V_s - L \cdot \frac{di_L(t)}{dt}$$

$$\textcircled{2} \leftarrow i_c(t) = i_L(t) - I_0.$$



diff ①,

$$\frac{dV_c(t)}{dt} = \frac{-L \cdot d^2 i_L(t)}{dt^2}$$

$$\frac{\dot{i}_c(t)}{C} = \frac{-L \cdot d^2 i_L(t)}{dt^2}$$

$$\frac{i_L(t) - I_0}{C} = \frac{-L \cdot d^2 i_L(t)}{dt^2}$$

$$\frac{I_0 - i_L(t)}{C} = L \cdot \frac{d^2 i_L(t)}{dt^2}$$

$\div LC,$

$$\frac{I_0 - i_L(t)}{LC} = \frac{d^2 i_L(t)}{dt^2}$$

$$\frac{d^2 i_L(t)}{dt^2} + \frac{i_L(t)}{LC} = \frac{I_0}{LC} \quad \textcircled{4}$$

The solution of eqn ④ with initial condition $i_L(t_1) = I_0$.

$$\boxed{i_L(t) = I_0 + \frac{V_s}{\omega_0} \sin \omega_0 (t - t_1)} \quad \textcircled{5}$$

$$Z_0 = \sqrt{\frac{L}{C}} ; \omega_0 = \frac{1}{\sqrt{LC}}$$

Eqn (5) is valid until i_L reaches zero at $t = t_2$

The switch is turned off after the current reaches zero, resulting in zero current switching and no switching power loss.

$$i_L(t) = 0, \text{ at } t = t_2.$$

$$(5) \Rightarrow 0 = I_0 + \frac{V_s}{Z_0} \sin \omega_0 (t_2 - t_1).$$

$$\sin \omega_0 (t_2 - t_1) = -\frac{I_0 Z_0}{V_s}$$

$$t_2 - t_1 = \frac{1}{\omega_0} \sin^{-1} \left[-\frac{I_0 Z_0}{V_s} \right].$$

Sub (5) in (1).

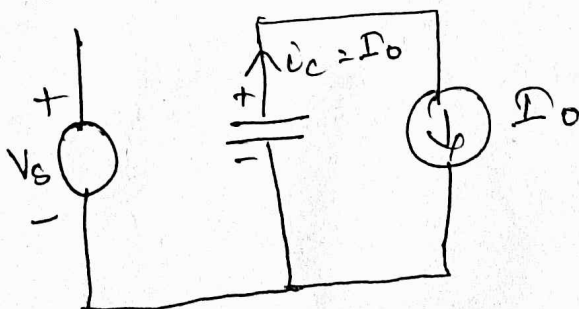
$$V_c(t) = V_s - L \frac{d}{dt} \left[I_0 + \frac{V_s}{Z_0} \sin \omega_0 (t - t_1) \right].$$

$$= V_s - L \cdot \frac{V_s}{Z_0} \omega_0 \cdot \cos \omega_0 (t - t_1).$$

$$= V_s \left[1 - \frac{L}{Z_0} \omega_0 \cdot \cos \omega_0 (t - t_1) \right].$$

$$V_c(t) = V_s \left[1 - \cos \omega_0 (t - t_1) \right].$$

Analysis for $t_2 \leq t \leq t_3$:-



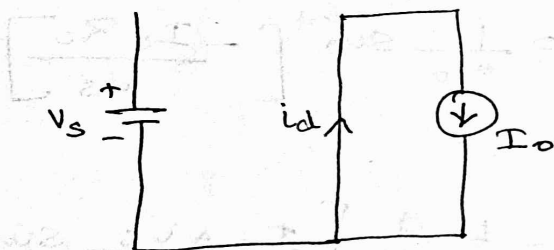
After the inductor current reaches zero at t_2 , the switch is opened. The diodes is off since $V_c > 0$. capacitor current is $-I_0$.

$$V_c(t) = \frac{1}{C} \int_{t_2}^t -I_0 \cdot dt + V_c(t_2).$$

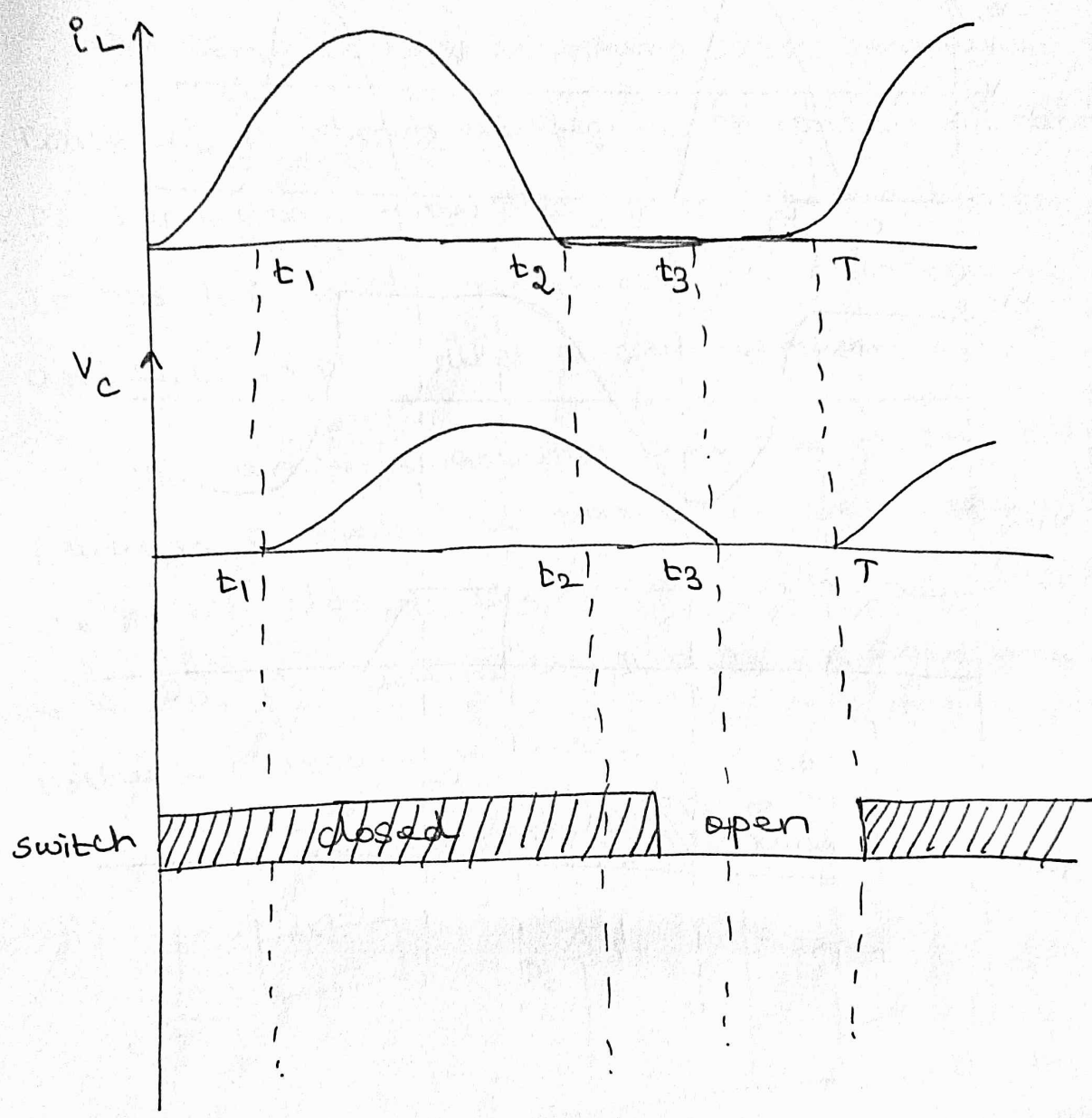
$$= -\frac{I_0}{C} (t_2 - t) + V_c(t_2).$$

$$V_c(t) = \frac{I_0}{C} (t - t_2) + V_c(t_2).$$

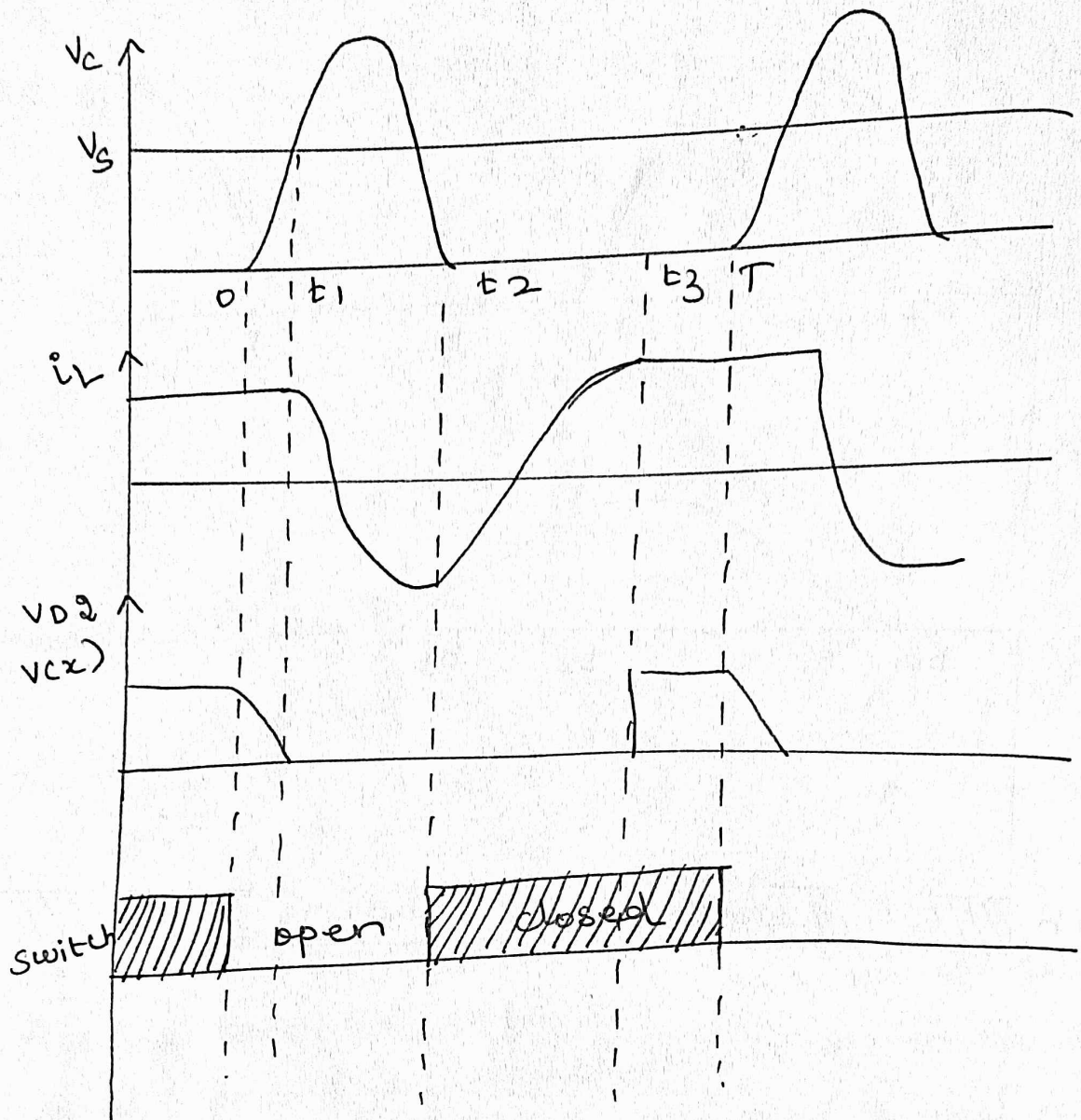
Analysis for $t_3 \leq t \leq T$:



If I_0 is constant, the capacitor voltage decreases linearly. When the capacitor voltage reaches zero, the diode becomes forward biased to carry I_0 . The circuit is then back at the starting point. The duration of this interval is the difference between the switching period T and the other time intervals which are determined from other circuit parameters.



ZCS \rightarrow waveform.

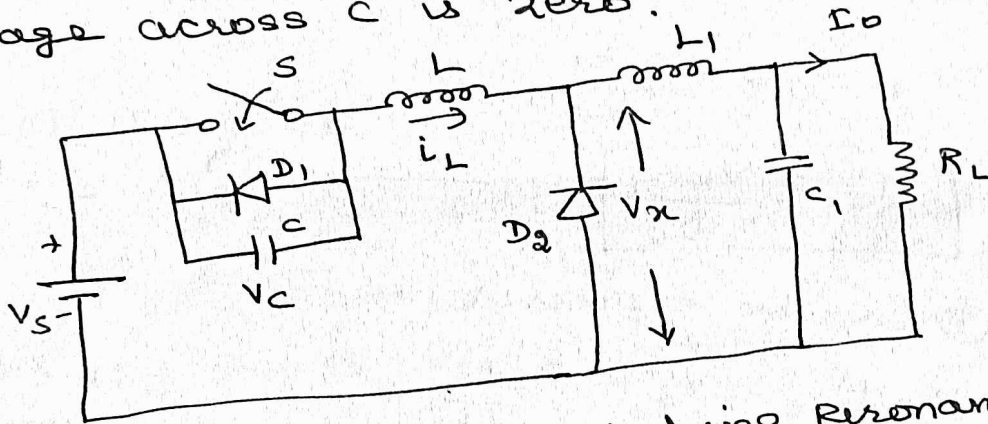


ZVS \rightarrow waveform.

Zero Voltage Switching Resonant Converter

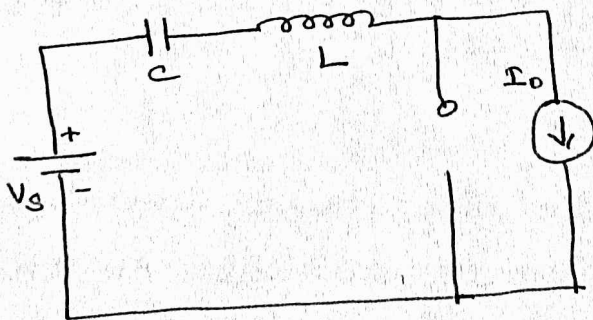
The zero voltage switching (ZVS) resonant converter is shown in figure. It consists of diode D_1 , capacitor C connected across the switch S . It has L, C as the resonant circuit components and L_1, C_1 as the filter circuit components.

The analysis assumes that the output filter produces a ripple free current I_0 . Beginning with the switch closed, the current in the switch and L is I_0 , the current in D_1 & D_2 are zero and the voltage across C is zero.



Zero Voltage Switching Resonant Converter

Analysis of $0 \leq t \leq t_1$



The switch is opened at $t=0$, The capacitor current is then I_0 , causing the capacitor voltage initially zero to increase linearly. The voltage across the capacitor C is,

$$V_C(t) = \frac{1}{C} \int I_0 \cdot dt$$

$$\boxed{V_C(t) = \frac{I_0 t}{C}} \quad \text{--- (1)}$$

The voltage across L is zero. The voltage at the filter input is

$$V_x(t) = V_S - V_C(t)$$

$$\boxed{V_x(t) = V_S - \frac{I_0 t}{C}} \quad \text{--- (2)}$$

At $t=t_1$, $V_x = 0$, the diode turns on.

$$0 = V_S - \frac{I_0 t_1}{C}$$

$$\boxed{t_1 = \frac{V_S \cdot C}{I_0}} \quad \text{--- (3)}$$

$$C = \frac{t_1 I_0}{V_S}$$

Eqn (2) can be expressed as,

$$V_x(t) = V_S - \frac{I_0 t}{\frac{I_0 t_1}{V_S}}$$

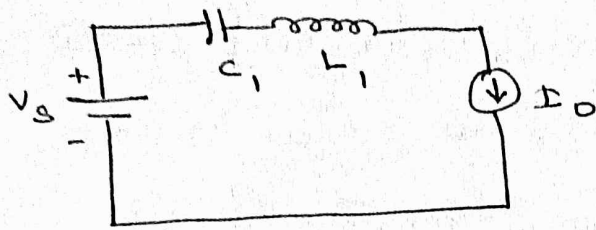
sub C value in eqn (2)

$$= V_S - V_S \times t/t_1$$

$$\boxed{V_x(t) = V_S [1 - t/t_1]} \quad \text{--- (4)}$$

Analysis for $t_1 \leq t \leq t_2$

When V_C reaches the source voltage V_S the diode D_2 becomes forward biased. At this time i_L and V_C in the circuit begin to oscillate.



$$L_1 \frac{di_{L_1}(t)}{dt} + V_{C_1}(t) = V_s.$$

Diff,

$$L_1 \frac{d^2 i_{L_1}(t)}{dt^2} + \frac{dV_{C_1}(t)}{dt} = 0 \quad \text{--- (5)}$$

$$\text{but } \frac{dV_{C_1}(t)}{dt} = \frac{i_C(t)}{C_1} \quad \text{--- (6)}$$

sub (6) in (5),

$$L_1 \frac{d^2 i_{L_1}(t)}{dt^2} + \frac{i_C(t)}{C_1} = 0.$$

$i_C(t) = i_L(t)$ since both are in series.

$$\frac{d^2 i_{L_1}(t)}{dt^2} + \frac{i_{L_1}(t)}{L_1 C_1} = 0.$$

Solving for $i_L(t)$ with initial condition,

$$i_L(t_1) = I_0.$$

$$\boxed{i_L(t) = I_0 \cos[\omega_0(t-t_1)]} \quad \text{--- (7)}$$

$$\omega_0 = \frac{1}{\sqrt{L_1 C_1}}.$$

capacitor voltage is,

$$V_C(t) = \frac{1}{C_1} \int_{t_1}^t i_C(t) \cdot dt + V_C(t_1).$$

$$= \frac{1}{C_1} \int_{t_1}^t I_0 \cos[\omega_0(t-t_1)] dt + V_s$$

$$V_c(t) = V_s + I_0 Z_0 \sin[\omega_0(t-t_1)] \quad \text{--- (8)}$$

$$Z_0 = \sqrt{\frac{L}{C_1}}$$

At $t = t_2$, $V_c = 0$ from eqn (8)

~~t_2~~ \Rightarrow

$$0 = V_s + I_0 Z_0 \sin[\omega_0(t_2-t_1)]$$

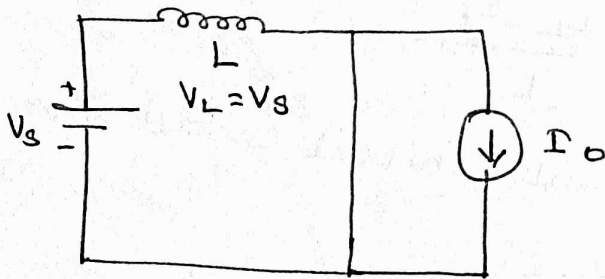
$$I_0 Z_0 \sin[\omega_0(t_2-t_1)] = -V_s$$

$$\sin[\omega_0(t_2-t_1)] = \frac{-V_s}{I_0 Z_0}$$

$$\omega_0(t_2-t_1) = \sin^{-1}\left[\frac{-V_s}{I_0 Z_0}\right]$$

$$t_2 = \frac{1}{\omega_0} \left[\sin^{-1}\left(\frac{-V_s}{I_0 Z_0}\right) \right] + t_1 \quad \text{--- (9)}$$

Analysis of $t_2 \leq t \leq t_3$



After t_2 , both diodes are forward biased, the voltage across L_1 is V_s , i_L increases linearly until it reaches I_0 at t_3 . The switch is closed after t_2 when $V_c = 0$ and the diode is ON to carry a negative current i_L . The current $i_L(t)$ in the interval from t_2 to t_3 is expressed as,

$$i_L(t) = \frac{1}{L_1} \int_{t_2}^t V_s \cdot dt + i_L(t_2)$$

$$i_L(t) = \frac{V_s}{L_1} (t - t_2) + I_0 \cos [\omega_0 (t_2 - t_1)] \quad \text{--- (10)}$$

current at t_3 is I_0 ,

$$i_L(t_3) = I_0$$

$$I_0 = \frac{V_s}{L_1} (t_3 - t_2) + I_0 \cos [\omega_0 (t_2 - t_1)]$$

$$t_3 = \frac{L_1 I_0}{V_s} [1 - \cos (\omega_0 (t_2 - t_1))] + t_2$$

At $t = t_3$, diode D_2 turns on,

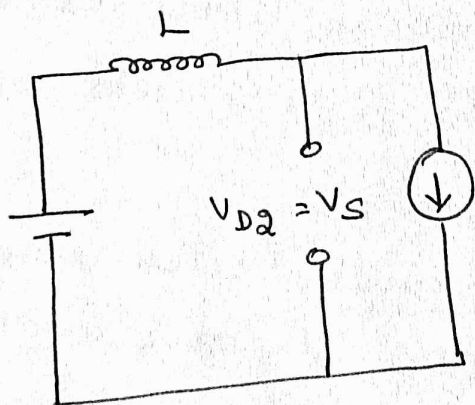
hence $V_x = 0$.

Analysis for $t_3 \leq t \leq T$

In this interval, the switch is closed both diodes are off, the current in the switch is I_0 . The circuit in this condition until the switch is reopened.

$$V_x = V_s$$

The time interval $T - t_3$ is determined by the switching frequency of the circuit.



Battery operated vehicles.

A battery electric vehicle (BEV) is a pure electric vehicle, that uses chemical energy stored in rechargeable battery packs. BEV use electric motors and motor controllers instead of internal combustion engines (ICEs) for propulsion.

Motor controllers :-

The motor controller receives a signal from potentiometers linked to the accelerator pedal. This DC power is supplied by the battery pack and the controller regulates the power to the motor supplying either variable pulse width DC or variable frequency variable amplitude AC, depending on the motor type.

Battery Pack :-

Lithium-ion batteries which are found in most EVs today. The battery pack powering modern EVs can have as little as 96 battery cells to as many as 2,976 cells.

Motors :

Electric cars used series wound DC motors a form of brushed DC electric motor. Most recent electric vehicles use a variety of AC motor types. There are usually induction motors or brushless AC electric motors which use permanent magnets.

Environmental benefits of the use of electric

Vehicles:

Electric vehicles produce no GHG emissions. The two factors driving these GHG emissions of

BEV are:

i) The carbon intensity of the electricity used to recharge the EV.

ii) Consumption of the specific vehicle (km/kwh)

Electric car saved 50% - 60% of CO₂ emissions compared to diesel and gasoline fuelled engines.

Disadvantages :-

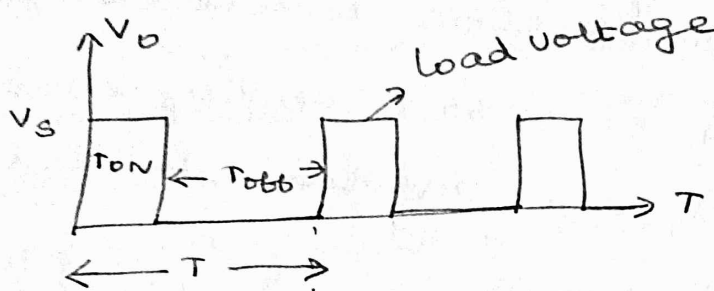
- 1) Carbon dioxide is still emitted when the electric vehicle is being manufactured.
- 2) The lithium-ion batteries used in the vehicle take more materials and energy to produce because of the extraction process of the lithium and cobalt essential to the battery.
- 3) Bigger the EV, more carbon dioxide emitted.
- 4) The mines are used to produce lithium and cobalt used in the battery are also creating problems for environment, creating health problems for the animals and people that live nearby.

Control strategies :-

The various control strategies for varying duty cycle α are as follows :-

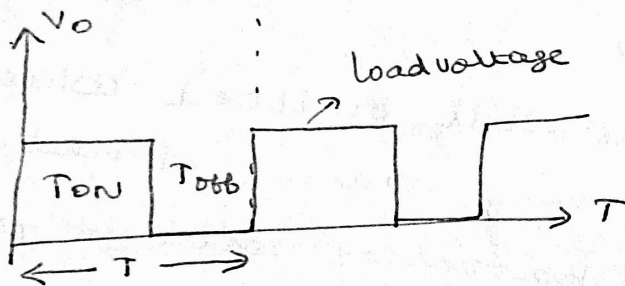
i) constant frequency system : (Time ratio control)

The on time T_{on} is varied but chopping frequency f (or chopping period T) is kept constant. The pulse width is adjusted and variation of T_{on} occurred. This is known as pulse-width modulation scheme.



$$T_{on} = \frac{1}{4} T$$

$$\alpha = 0.25 \text{ or } 25\%$$



$$T_{on} = \frac{3}{4} T,$$

$$\alpha = 0.75 \text{ or } 75\%$$

(ii) Variable frequency system :-

The chopping frequency f (chopping period T) is varied and either on-time T_{on} kept constant or off-time T_{off} is kept constant. The method of controlling α is also called frequency modulation scheme.

Disadvantages of frequency modulation scheme as compared to pulse-width modulation :

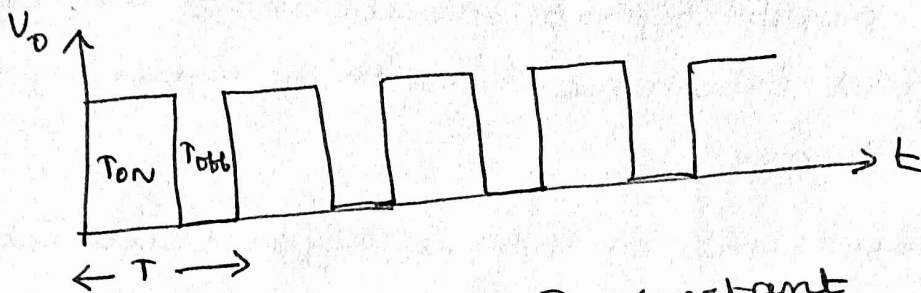
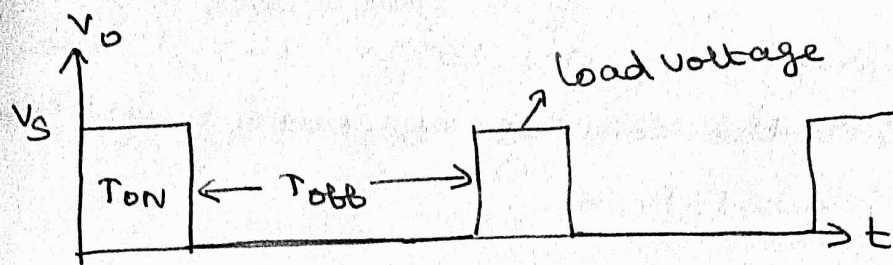


fig: (a) on-time T_{on} constant

$T_{on} \rightarrow$ constant, but T is varied.

$$T_{on} = \frac{1}{4} T, \quad \alpha = 0.25$$

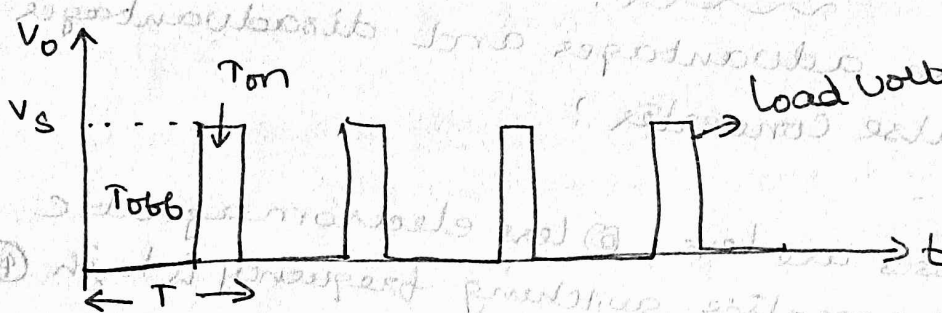
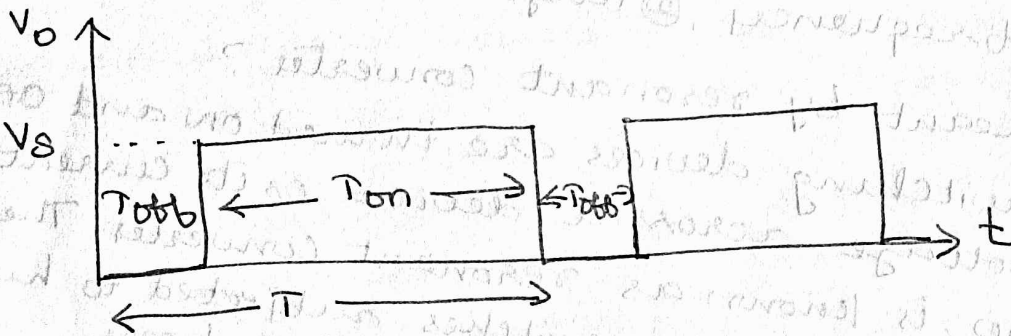


fig: (b) off time T_{off} constant



$$T_{on} = \frac{3}{4} T, \quad \alpha = 0.75$$

$T_{off} \rightarrow$ constant, T varied

fig: off time T_{off} constant

5/10/20

switching Mode Regulators

DC converters can be used as switching mode regulators to convert a dc voltage, normally unregulated to a regulated dc output voltage.

There are four basic topologies of switching Regulators :

1. Buck Regulators
2. Boost Regulators
3. Buck-Boost Regulators
4. Cuk Regulators.

Buck Regulator :- [step down converter]

The average output voltage V_a is less than the input voltage V_s , hence the name buck, a popular regulator.

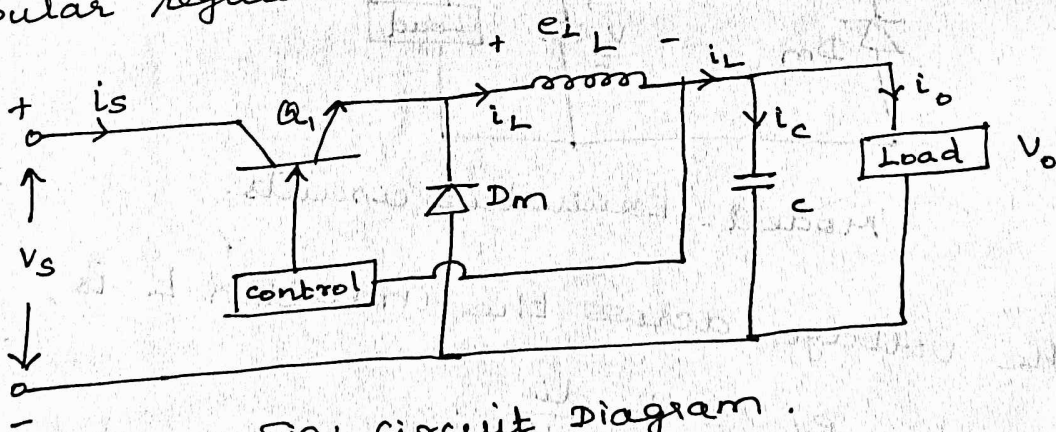
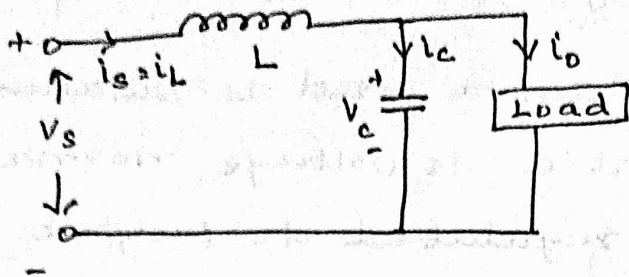


Fig: circuit diagram.

The circuit diagram of a buck regulator using a Power BJT is shown in fig. circuit operation can be divided into two modes.

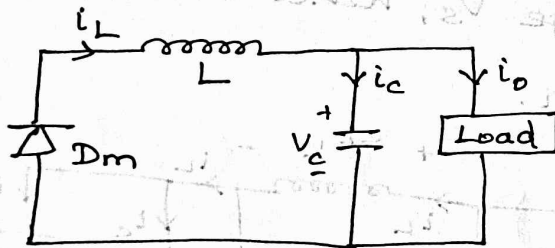
Mode I : Mode I begins when transistor Q_1 is switched on at $t = 0$.



Mode 1 - Equivalent circuit

The input current which rises, flows through filter inductor \$L\$, filter capacitor \$C\$, load Resistor \$R\$.

Mode 2 : Mode 2 begins when transistor \$Q_1\$ is switched off at \$t = t_1\$. The freewheeling Diode \$D_m\$ conducts due to energy stored in the inductor. The inductor current continues to flow through \$L\$, \$C\$, load and diode \$D_m\$.



Mode 2 - Equivalent circuits.

The voltage across the inductor \$L\$ is,

$$e_L = L \cdot \frac{di}{dt}$$

Assuming inductor current rises linearly from \$I_1\$ to \$I_2\$, in time \$t_1\$.

$$V_s - V_a = L \cdot \frac{(I_2 - I_1)}{t_1} \quad \text{--- (1)}$$

$$= L \cdot \frac{\Delta I}{t_1}$$

$\Delta I = I_2 - I_1$

\$\Delta I \rightarrow\$ Peak to peak ripple current of inductor

$$t_1 = \frac{\Delta I \cdot L}{V_s - V_a} \quad \text{--- (2)}$$

inductor current falls linearly from I_2 to I_1 in time t_2 ,

from eqn (1),

$$-V_a = -L \cdot \frac{\Delta I}{t_2}$$

$$[\because \text{at } t_1 \rightarrow \text{off, } V_s = 0]$$

$$t_2 = \frac{\Delta I \cdot L}{V_a} \quad \text{--- (3)}$$

Equating the value of ΔI in eqns (2) & (3).

$$\textcircled{2} \Rightarrow \Delta I = \frac{(V_s - V_a) t_1}{L}$$

$$\textcircled{3} \Rightarrow \Delta I = \frac{V_a t_2}{L}$$

$$\Delta I = \frac{(V_s - V_a) t_1}{L} = \frac{V_a t_2}{L}$$

$$\Delta I = (V_s - V_a) t_1 = V_a t_2$$

sub $t_1 = kT$, $t_2 = (1-k)T$, the average output

voltage is, $(V_s - V_a) kT = V_a (1-k) T$.

$$V_s k - V_a k = V_a - V_a k$$

$$V_s k = V_a$$

$$\boxed{t_1 = kT}$$

$$k = \frac{t_1}{T}$$

$$\boxed{V_a = V_s \frac{t_1}{T} = k V_s} \quad \text{--- (4)}$$

Assuming a lossless circuit,

$$V_s I_s = V_a I_a$$

$$\text{Sub } V_a = k V_s.$$

$$V_s I_s = k V_s I_a.$$

$$\boxed{I_s = k I_a.} \quad \text{--- (5)}$$

switching period T can be expressed as,

$$T = \frac{1}{f} = t_1 + t_2 = \frac{\Delta I L}{V_s - V_a} + \frac{\Delta I L}{V_a}$$

(from (2) & (3))

$$= \frac{\Delta I \cdot L \cdot V_a + \Delta I \cdot L (V_s - V_a)}{V_a (V_s - V_a)}$$

$$= \Delta I \cdot L \cdot \frac{V_s}{V_a (V_s - V_a)}$$

$$= \frac{\Delta I \cdot L \cdot V_s}{V_a (V_s - V_a)}$$

$$\boxed{T = \frac{\Delta I \cdot L \cdot V_s}{V_a (V_s - V_a)}} \quad \text{--- (6)}$$

Peak to peak ripple current as,

$$\Delta I = \frac{V_a (V_s - V_a) T}{L \cdot V_s}$$

$T = \frac{1}{f}$

$$\boxed{\Delta I = \frac{V_a (V_s - V_a)}{f \cdot L \cdot V_s}}$$

--- (7)

Using Kirchhoff's current law,

$$i_L = i_c + i_o.$$

$$\Delta i_L = \Delta i_c + \Delta i_o.$$

$$\Delta i_L = \Delta i_c.$$

$\Delta i_o \rightarrow$ is very small, and negligible.

The average capacitor current flows into for,

$$\frac{t_1}{2} + \frac{t_2}{2} = T/2 \text{ is,}$$

$$I_c = \frac{\Delta I}{4}$$

capacitor voltage is expressed as,

$$V_c = \frac{1}{c} \int I_c \cdot dt + V_c(t=0)$$

Peak to peak ripple voltage of the capacitor is,

$$\begin{aligned} \Delta V_c &= V_c - V_c(t=0) \\ &= \frac{1}{c} \int_0^{T/2} \frac{\Delta I}{4} dt \end{aligned}$$

$$= \frac{1}{c} \frac{\Delta I}{4} [t]_0^{T/2}$$

$$\Delta V_c = \frac{1}{c} \frac{\Delta I}{4} \times T/2 = \frac{\Delta I \cdot T}{8c} = \frac{\Delta I}{8f c}$$

$$\boxed{\Delta V_c = \frac{\Delta I}{8f c}} \quad \text{--- (8)}$$

Sub the value of ΔI from (7) in eqn (8).

$$\boxed{\Delta V_c = \frac{V_a (V_s - V_a)}{8f^2 L C V_s}} \quad \text{--- (9)}$$

critical value of inductor $L_c = \frac{(1-k) R}{2f}$

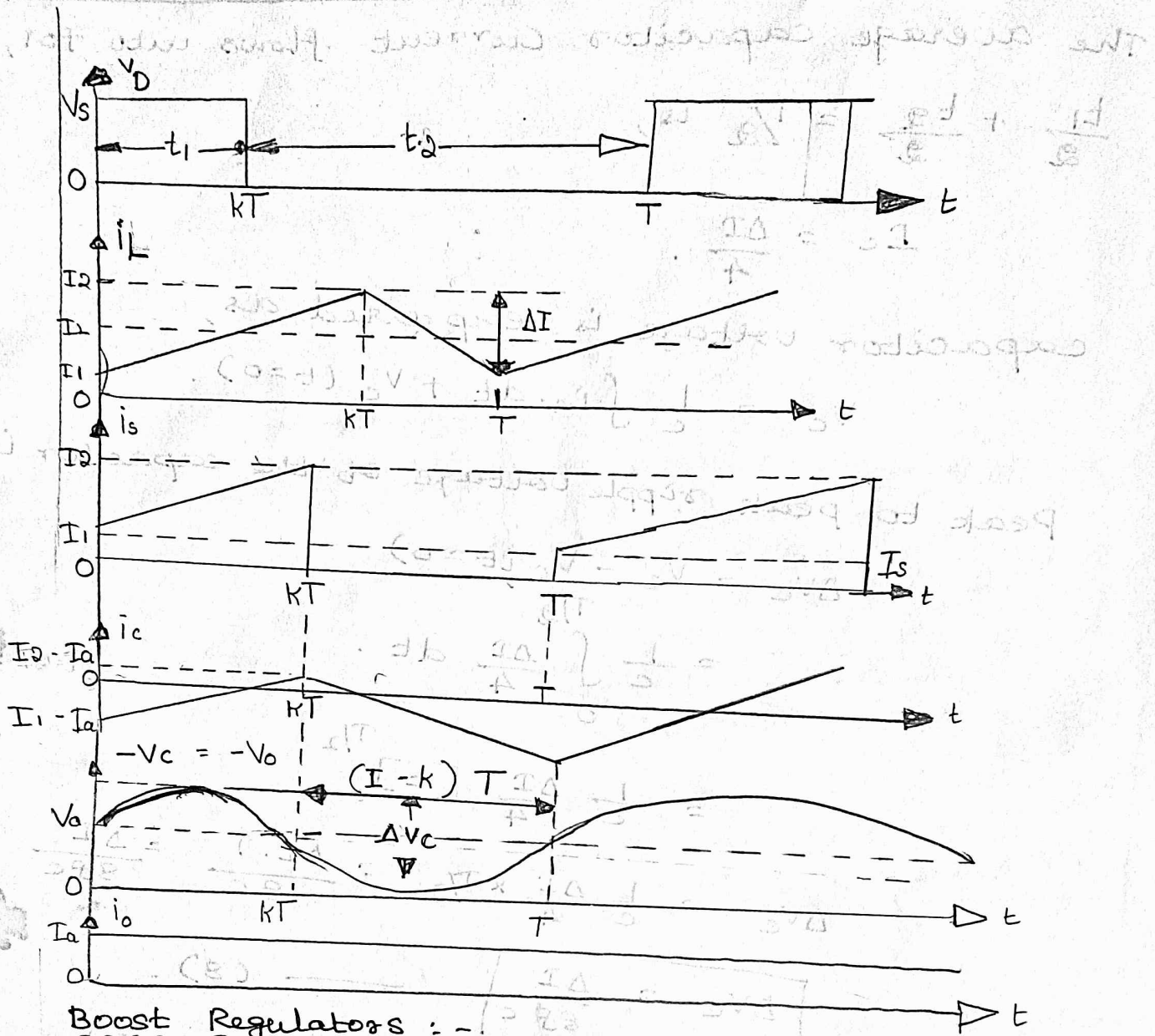
critical value of capacitor $C_c = \frac{1-k}{16L f^2}$

Advantages:

- ① It requires only one Transistor, simple, high efficiency greater than 90%, di/dt of load current limited by L.

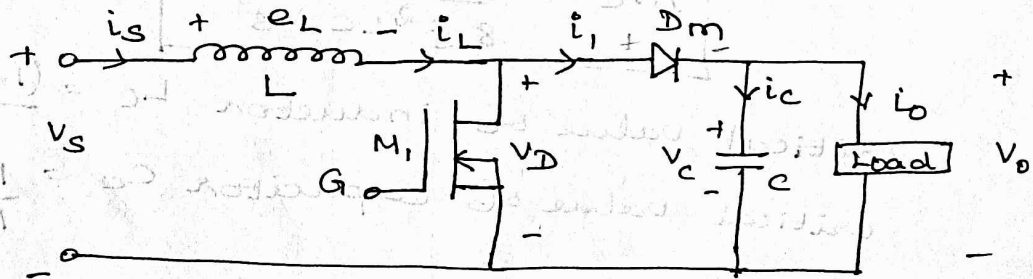
Disadvantages:

- ① Input current is discontinuous,
- ② smoothing filter required.
- ③ It provide one polarity of output voltage.
- ④ It requires a protection circuit.



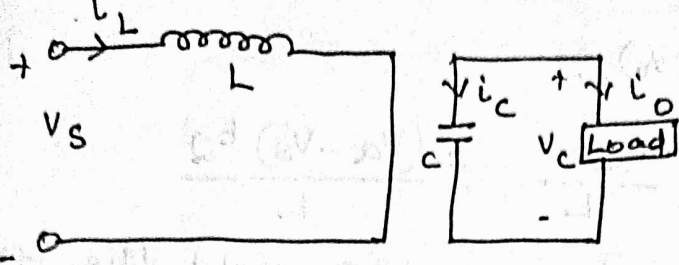
Boost Regulators :-

In a boost regulator, the output voltage is greater than the input voltage hence the name boost.



Circuit Diagram.

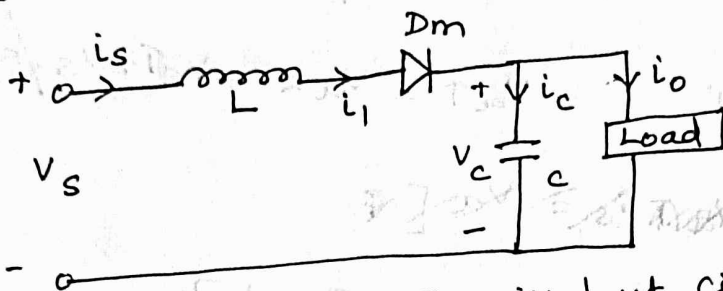
A boost regulator using a power MOSFET is shown in figure.



Mode-1 - Equivalent circuit

Model :

Mode 1 begins when transistor M_1 is switched on at $t=0$. The input current which rises, flows through inductor L & transistor Q_1 .



Mode 2 - Equivalent circuit.

Mode 2 begins when transistor M_1 is switched off at $t = t_1$. The current flow through L , C , load and diode D_m . The inductor current falls. The energy stored in inductor L is transferred to the load.

Assuming that the inductor current rises linearly from I_1 to I_2 in time t_1 .

$$V_s = L \frac{(I_2 - I_1)}{t_1} = L \cdot \frac{\Delta I}{t_1} \quad \text{--- (1)}$$

$$t_1 = \frac{L \cdot \Delta I}{V_s}$$

--- (2). $\Delta I \rightarrow$ peak to peak ripple current of inductor L .

The inductor current falls linearly from I_2 to I_1 in time t_2 ,

$$\frac{V_a - V_s}{V_s - V_a} = -L \cdot \frac{\Delta I}{t_2} \quad \text{--- (3)}$$

$$t_2 = \frac{+L \cdot \Delta I}{V_a - V_s} \quad \text{--- (4)}$$

From (2) & (4),

$$\Delta I = \frac{V_s t_1}{L} = \frac{(V_a - V_s) t_2}{L}$$

sub $t_1 = kT$, $t_2 = (1-k)T$ yield the average output voltage,

$$\frac{V_s kT}{L} = \frac{(V_a - V_s)(1-k)T}{L}$$

$$\frac{V_s kT}{L} = \frac{V_a T - V_a kT - V_s T + V_s kT}{L}$$

$$V_s kT = V_a T - V_a kT - V_s T + V_s kT$$

~~$$V_s = V_a$$~~

$$V_s T = V_a T [1-k]$$

$$V_a = \frac{V_s}{1-k} \quad \text{--- (5)}$$

sub $k = \frac{t_1}{T} = t_1 f$ in eqn (5)

$$V_a = \frac{V_s}{1-t_1 f}$$

Assuming a lossless circuit, $V_s I_s = V_a I_a$

$$\frac{V_s I_s}{1-t_1 f} = \frac{V_s I_a}{1-t_1 f}$$

$$I_s = \frac{I_a}{1-t_1 f} \quad \text{--- (6)}$$

The switching period T can be found from,

$$T = \frac{1}{f} = t_1 + t_2$$

$$= \frac{\Delta I L}{V_s} + \frac{\Delta I L}{V_a - V_s}$$

$$T = \frac{\Delta I L V_a}{V_s (V_a - V_s)}$$

$$= \frac{\Delta I L (V_a - V_s) + \Delta I L V_s}{V_s (V_a - V_s)}$$

$$T = \frac{\Delta I L V_a - \Delta I L / V_s + \Delta I L V_s}{V_s (V_a - V_s)}$$

Peak to peak ripple current:

$$\Delta I = \frac{V_s (V_a - V_s) T}{L \cdot V_a}$$

$$\Delta I = \frac{V_s (V_a - V_s)}{f \cdot L \cdot V_a} \quad \text{--- (7)} \quad \because T = 1/f$$

When the transistor is on, the capacitor supplies the load current for $t = t_1$. The average capacitor current during time t_1 is $I_c = I_a$.

Peak to peak ripple voltage of the capacitor is,

$$\Delta V_c = \frac{1}{C} \int_0^{t_1} I_c \cdot dt$$

$$= \frac{1}{C} \int_0^{t_1} I_a \cdot dt = \frac{I_a t_1}{C}$$

$$\text{sub } t_1 = \frac{V_a - V_s}{V_a f}$$

$$\Delta V_c = \frac{I_a (V_a - V_s)}{C \cdot V_a \cdot f}$$

$$\Delta V_c = \frac{I_a K}{f C} \quad \text{--- (8)}$$

Condition for ~~maximum~~ continuous inductor current and capacitor voltage:

$$L_c = \frac{k(1-k)R}{2f}$$

$$C_c = \frac{k}{2fR}$$

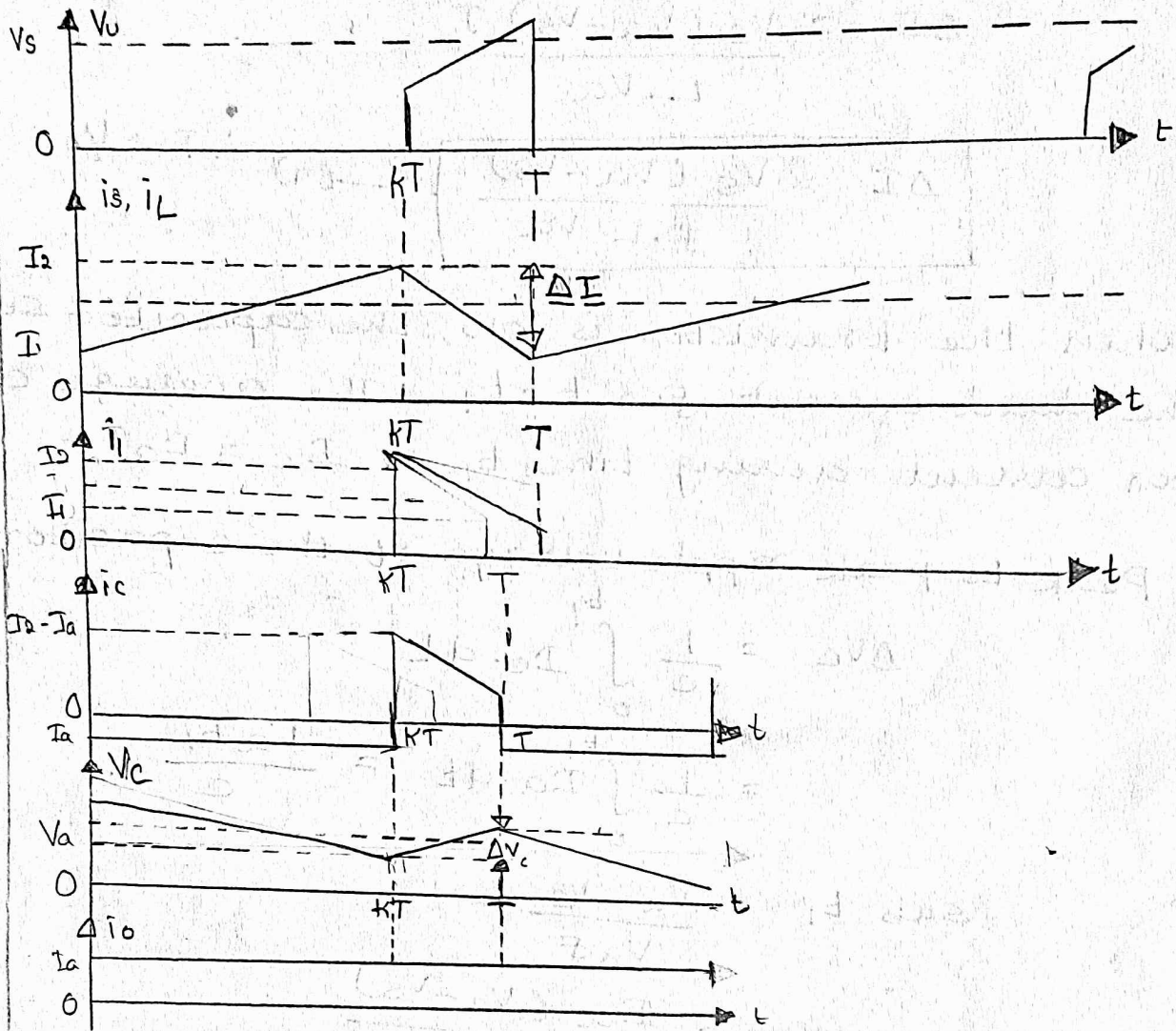
Advantages:

- ① Step up the output voltage without a Transformer.
- ② High Efficiency. Input current continuous.

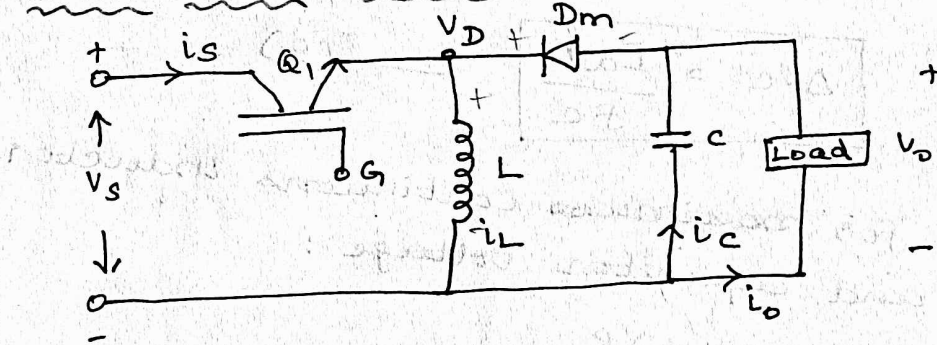
Disadvantages: -

- ① High peak current has to flow through the transistor.
- ② Difficult to stabilize the regulator.
- ③ Larger filter capacitor and inductor are required.

Buck - Boost Regulators:



Buck - Boost Regulators :

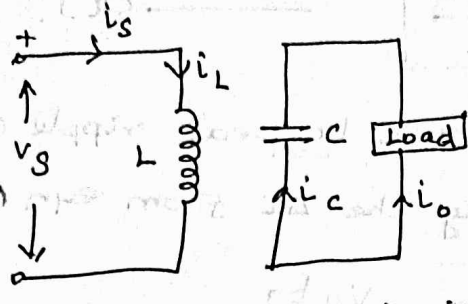


A Buck Boost regulator provides an output voltage that may be less than or greater than the input voltage - hence the name buck-boost. The output voltage polarity is opposite to that of the input voltage.

This regulator is also known as an inverting regulator.

Mode I :-

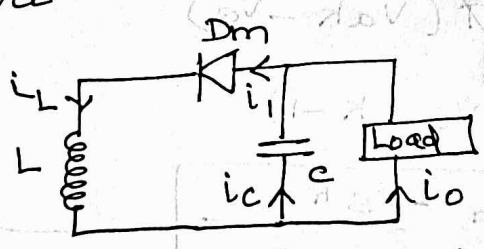
During mode 1, transistor Q_1 is turned on and diode D_m is reverse biased. The input current which rises, flows through inductor L , transistor Q_1 .



Equivalent circuit

Mode II :-

During mode 2, transistor Q_1 is switched off and the current which was flowing through inductor L , C , D_m and the load. The energy stored in L would be transferred to the load and inductor current would fall.



Equivalent circuit.

Assuming the inductor current rises linearly from I_1 to I_2 in time t_1 .

$$V_s = L \frac{(I_2 - I_1)}{t_1} = L \cdot \frac{\Delta I}{t_1}$$

$$t_1 = \frac{L \cdot \Delta I}{V_s} \quad \text{--- (1)}$$

and the inductor current falls linearly from I_2 to I_1 in time t_2 .

$$V_a = -L \cdot \frac{\Delta I}{t_2}$$

$$\boxed{t_2 = \frac{-\Delta I \cdot L}{V_a}} \quad \text{--- (2)}$$

$\Delta I = I_2 - I_1$ is the peak to peak ripple current of inductor L . Equating the ΔI from eqn (1) & (2).

$$\Delta I = \frac{V_s t_1}{L} = \frac{-V_a t_2}{L}$$

sub $t_1 = kT$, $t_2 = (1-k)T$, the average output voltage is

$$V_s(kT) = -V_a(1-k)T$$

$$= -V_a(T - kT)$$

$$V_s kT = -V_a T + V_a kT$$

$$V_s kT = T(V_a k - V_a)$$

$$V_s k = V_a(k - 1)$$

$$\boxed{V_a = \frac{-V_s k}{(1-k)}} \quad \text{--- (3)}$$

Assuming a lossless circuit, $V_s I_s = -V_a I_a$

$$V_s I_s = \frac{V_s k \cdot I_a}{(1-k)}$$

$$\boxed{I_s = \frac{I_a \cdot k}{1-k}} \quad \text{--- (4)}$$

The switching period T , can be found from,

$$T = \frac{1}{f} = t_1 + t_2$$

$$= \frac{\Delta I L}{V_s} + \left(-\frac{\Delta I L}{V_a} \right)$$

$$= \frac{\Delta I L V_a - \Delta I L V_s}{V_s V_a}$$

$$T = \frac{\Delta I L [V_a - V_s]}{V_s V_a}$$

$$\Delta I = \frac{T \cdot V_s V_a}{L \cdot (V_a - V_s)}$$

$$\Delta I = \frac{V_s \cdot V_a}{f \cdot L \cdot (V_a - V_s)} \quad \text{--- (5)}$$

When transistor Q_1 is on the filter capacitor supplies the load current for $t = t_1$. The average discharging current of the capacitor is $I_c = I_a$.

Peak to peak ripple voltage of the capacitor is,

$$\Delta V_c = \frac{1}{C} \int_0^{t_1} I_c \cdot dt = \frac{1}{C} \int_0^{t_1} I_a \cdot dt$$

$$\Delta V_c = \frac{I_a t_1}{C} \quad \text{--- (6)}$$

$$\Delta V_c = \frac{I_a V_a}{C_f (V_a - V_s)} \quad \text{--- (7)}$$

Condition for continuous inductor current and capacitor voltage:

$$I_c = \frac{(1-k)R}{2f}$$

critical value of the capacitor C_c

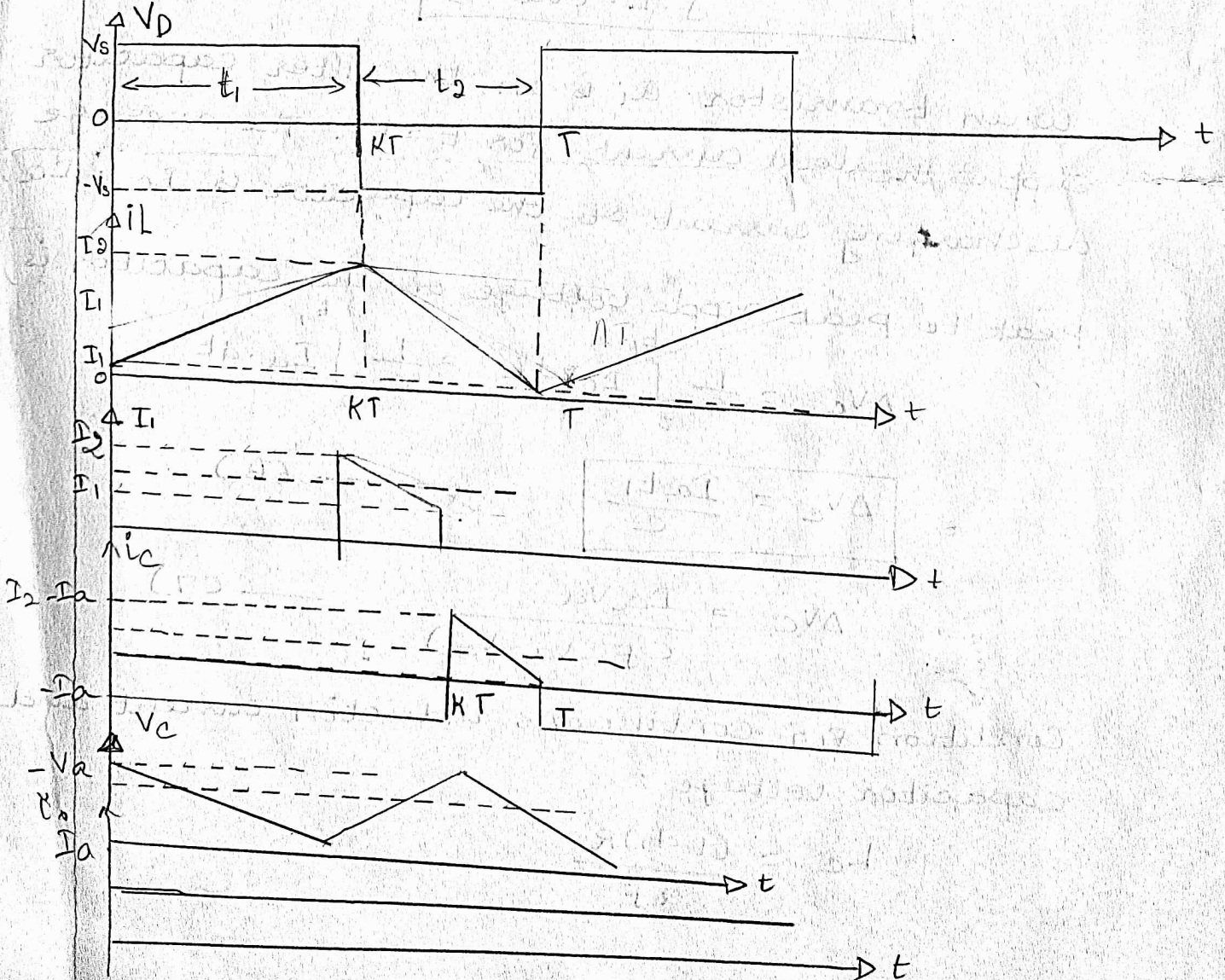
$$C_c = \frac{K}{2fR}$$

Advantages :

- 1) It provides output voltage polarity reversal without a transformer.
- 2) High Efficiency.
- 3) under a fault condition, the di/dt of the fault current is limited by the inductor L .
- 4) output short circuit protection easy to implement

Disadvantages :

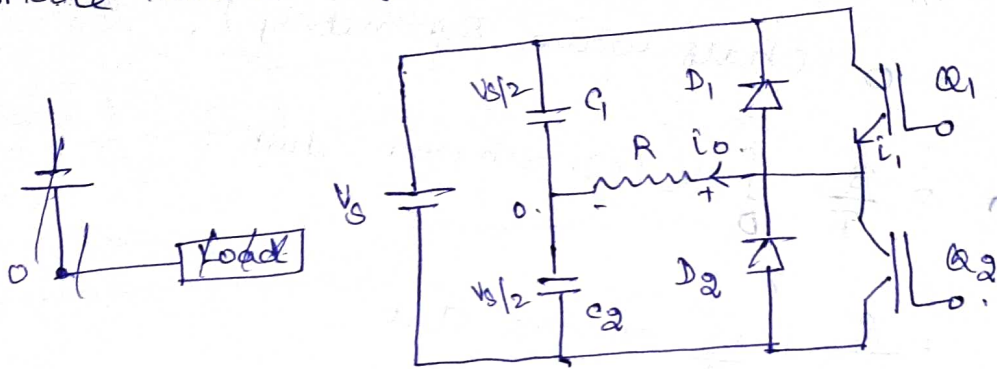
- 1) Input current is discontinuous.
- 2) high peak current flows through transistor Q_1 .



UNIT-IV

Inverters

Single phase Half Bridge Voltage source Inverters :-



- ⇒ The inverter circuit consists of two choppers, when transistor Q_1 is turned on, for a time $T_0/2$, instantaneous voltage across the load is $V_0 = V_s/2$.
- ⇒ If transistor Q_2 is turned on, for a time $T_0/2$ to T , $-V_s/2$ appears across a load.

⇒ Q_1, Q_2 are not turned on at the same time.

rms output voltage can be found from

$$V_0 = \left[\frac{2}{T_0} \int_0^{T_0/2} \frac{V_s^2}{4} dt \right]^{1/2}$$

$$= V_s/2$$

Instantaneous output voltage can be expressed as, in fourier series

$$V_0 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n (\cos n\omega t) + b_n \sin(n\omega t) \quad \text{--- (1)}$$

$$a_0 = \frac{2}{T} \int_0^T V_0(t) \cdot dt = \frac{2}{T} \int_0^{\pi} V_0(\omega t) \cdot d\omega t$$

$a_0 = 0$ (half wave symmetry).

$$a_n = \frac{2}{T} \int_0^T v_o(t) \cdot \cosh n\omega t \cdot dt$$

$$= \frac{2}{T} \int_0^T v_o(t) \cdot \cosh n\omega t \cdot dt$$

$$a_n = 0 \quad (\text{half wave symmetry})$$

$$b_n = \frac{2}{T} \int_0^T v_o(t) \cdot \sin n\omega t \cdot dt$$

$$= \frac{2}{T} \int_0^T \frac{V_s}{2} \sin n\omega t \cdot dt$$

$$= \frac{V_s}{T} \left[-\frac{\cos n\omega t}{n} \right]_0^T$$

$$= \frac{V_s}{nT} [-\cos n\pi + \cos 0]$$

$$= \frac{V_s}{nT} [\cos 0 - \cos n\pi]$$

$$\begin{aligned} \cos 0 &= 1 \\ \cos n\pi &= -1 \end{aligned}$$

$$\text{When } n=1, \quad = \frac{V_s}{T} [\cos 0 - \cos \pi] = \frac{2V_s}{T}$$

$$n=2, \quad = 0$$

$$n=3, \quad = \frac{2V_s}{3T}$$

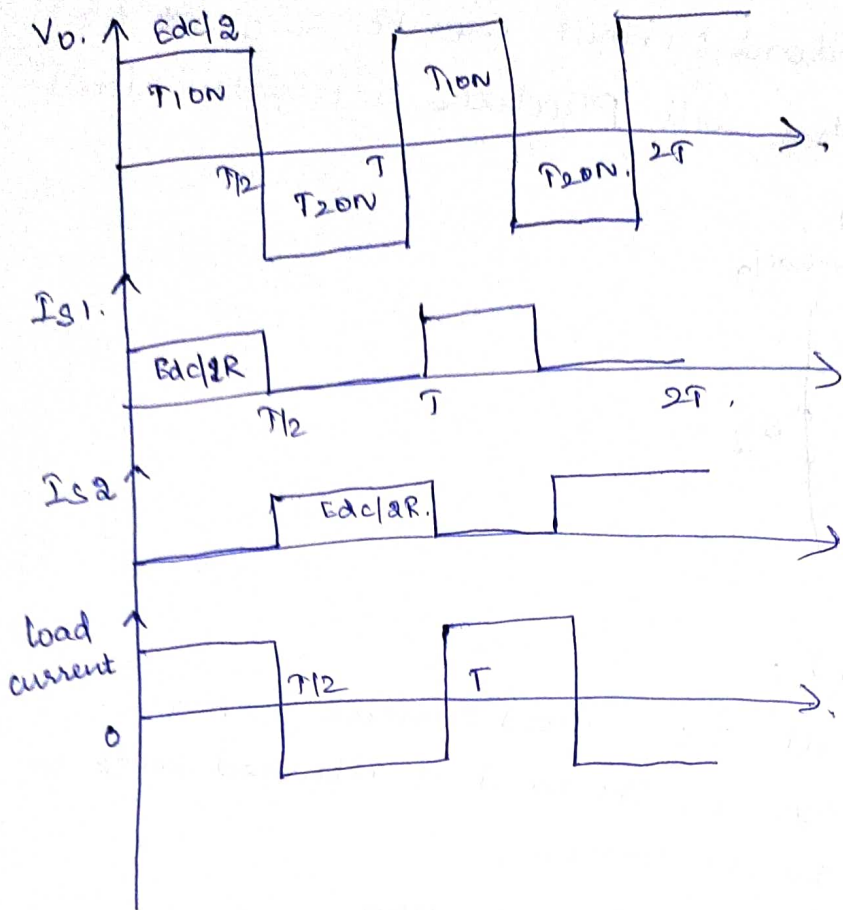
sub in eqn (1)

$$\therefore v_o(t) = \sum_{n=1,3,5} \frac{2V_s}{nT} \sin n\omega t$$

$$I_{T \text{ avg}} = \frac{1}{T} \int_0^{\pi/2} \frac{E_d c}{2R} dt = \frac{E_d c}{4R}$$

$$I_{T \text{ rms}} = \frac{E_d c}{2\sqrt{2} R}$$

$$I_{T \text{ peak}} = \frac{E_d c}{2R}$$

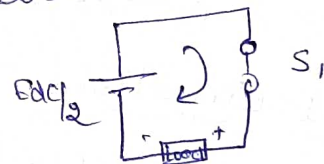


operation with RL load :-

⇒ with an inductive load, the output voltage waveform is similar to that with a R-load, but load current cannot change immediately.

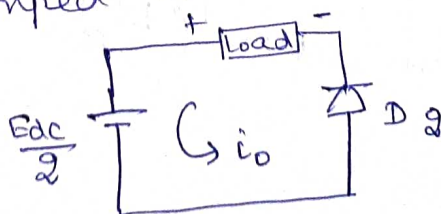
Mode 1 : ($t_1 < t < t_2$)

S_1 turned on at t_1 , load voltage = $E_{dc}/2$.
 at instant t_2 , load current reaches peak value.
 S_1 is turned off.



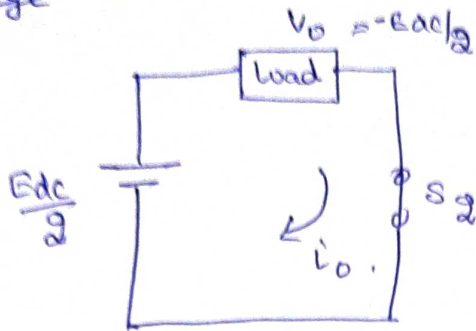
Mode 2 : ($t_2 < t < t_3$)

Due to inductive load, load current direction will be maintained even when S_1 is off. stored energy in load is fed back to the lower half of the source
 load voltage is clamped to $-E_{dc}/2$.



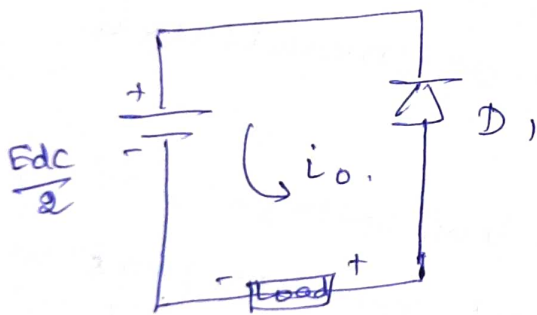
Mode 3 ($t_3 < t < t_4$).

at instant t_3 , load current goes to zero, at t_3 , S_2 is turned on. This will produce a negative load voltage $e_o = -E_{dc}/2$.



Mode 4: ($t_4 < t < t_5$):

S_2 turned off at t_4 , load current remains negative, stored energy in the load is returned back to the upper half of the dc source, at t_5 , load current goes to zero, S_1 turned on again.



Circuit Equations:

Instantaneous current (i_o)

$$i_o(t) = \sum_{n=1,3,5,\dots}^{\infty} \frac{2E_{dc}}{n\pi \sqrt{R^2 + (n\omega L)^2}} \sin(n\omega t - \theta_n).$$

$$Z_n = \sqrt{R^2 + (n\omega L)^2}.$$

$$\theta_n = \tan^{-1} \left(\frac{n\omega L}{R} \right).$$

When α is turned off at $t = T_0$, load current flows through D_1 , load, upper half of the d.c source.

When diode D_1 or D_2 conducts, energy is fed back to the source and these diodes are known as feedback diodes.

For an RL load, the instantaneous load current i_o is dividing the instantaneous output voltage by the load impedance $Z = R + j\omega L$.

$$i_o = \sum_{n=1,3,5}^{\infty} \frac{2V_s}{n\pi \sqrt{R^2 + (\omega L)^2}} \sin(n\omega t - \alpha_n)$$

$$\alpha_n = \tan^{-1} \frac{\omega L}{R}$$

Performance Parameters :-

(i) Harmonic factor of n th harmonic (HF_n):

$$HF_n = \frac{V_{on}}{V_{o1}} \quad \text{for } n > 1.$$

It is a measure of individual harmonic contribution.

$V_1 \rightarrow$ rms value of the fundamental component.

$V_{on} \rightarrow$ rms value of the n th harmonic component.

(ii) Total Harmonic Distortion (THD) :-

It is a measure of closeness in shape between a waveform and its fundamental component.

$$THD = \frac{1}{V_{o1}} \left[\sum_{n=2,3}^{\infty} V_{on}^2 \right]^{1/2}$$

(iii) Distortion factor (DF) :-

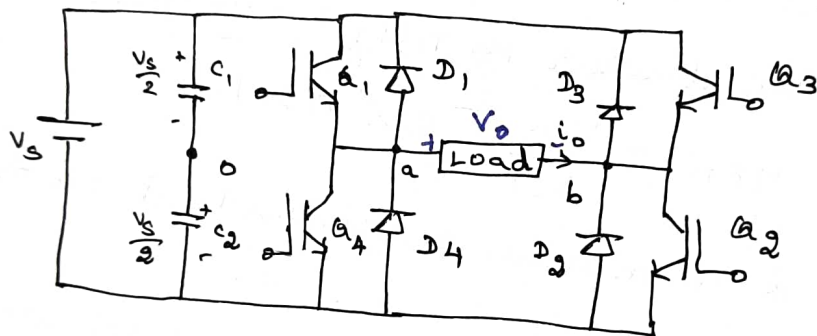
It indicates the amount of HD that remains in a particular waveform after the harmonics of that waveform have been subjected to a second order attenuation.

$$DF = \frac{1}{V_{01}} \left[\sum_{n=2,3,\dots}^{\infty} \left(\frac{V_{0n}}{n^2} \right)^2 \right]^{\frac{1}{2}}$$

iv) Lowest order harmonic (LOH) :

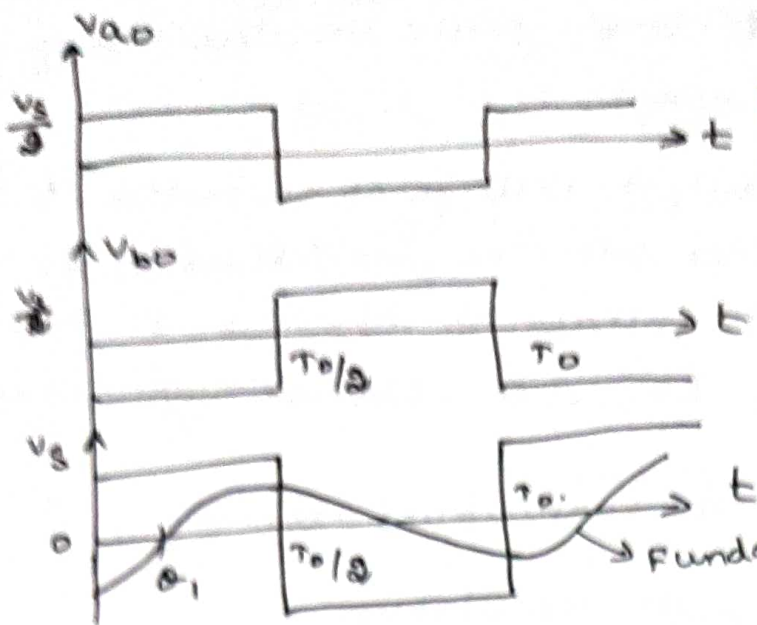
The LOH is that harmonic component whose frequency is closest to the fundamental one. Its amplitude is greater than or equal to 3% of the fundamental component.

Single phase Bridge Inverters :-



It consists of four choppers. When transistors Q_1 & Q_3 are turned ON simultaneously, the input voltage V_s appears across the load.

If transistors Q_3 & Q_4 are turned ON, the voltage across the load is reversed, and is $-V_s$.



$V_o = V_a - V_b$

switch state	V_a	V_b	$V_o = V_a - V_b$
Q_1, Q_2 ON	$\frac{V_s}{2}$	$-\frac{V_s}{2}$	$V_o = V_s$
Q_3, Q_4 ON	$-\frac{V_s}{2}$	$\frac{V_s}{2}$	$V_o = -V_s$
Q_1, Q_3 ON	$\frac{V_s}{2}$	$\frac{V_s}{2}$	$V_o = 0$
Q_2, Q_4 ON	$-\frac{V_s}{2}$	$-\frac{V_s}{2}$	$V_o = 0$

The rms output voltage can be found from,

$$V_o = \left(\frac{2}{T_o} \int_0^{T_o/2} V_s^2 dt \right)^{1/2} = V_s$$

Instantaneous output voltage in a fourier series,

$$V_o = \sum_{n=1,3,5}^{\infty} \frac{AV_s}{n\pi} \sin n\omega t$$

$$\begin{aligned}
 a_0 &= a_n = 0 \\
 b_n &= \frac{2}{\pi} \int_0^{\pi} V_o(t) \sin n\omega t d\omega t \\
 &= \frac{2}{\pi} \int_0^{\pi} V_s \sin n\omega t d\omega t \\
 &= \frac{2V_s}{n\pi} \left[-\frac{\cos n\omega t}{n} \right]_0^{\pi} \\
 &= \frac{2V_s}{n\pi} [\cos 0 - \cos \pi] \\
 &= \frac{4V_s}{n\pi}
 \end{aligned}$$

rms value of fundamental component

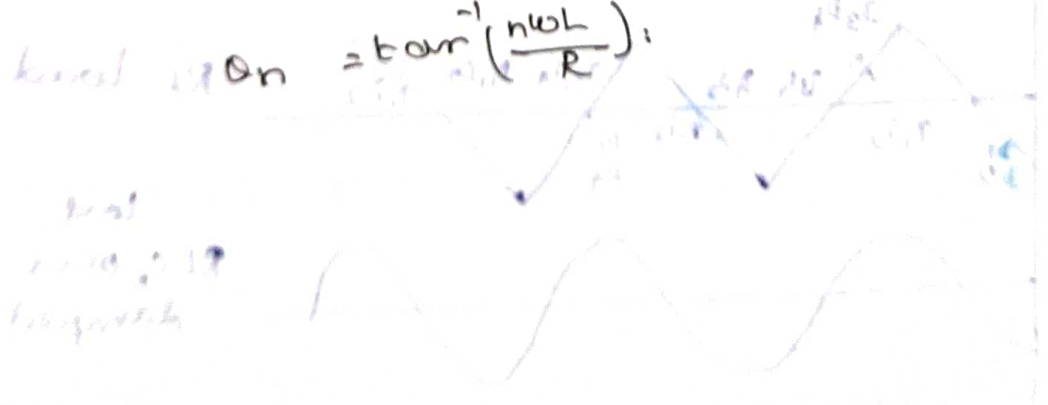
for $n=1$,

$$V_o = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{\sqrt{2}\pi} = 0.90 V_s$$

Instantaneous load current i_o for an RL load,

$$i_o = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{n\pi \sqrt{R^2 + (n\omega L)^2}} \sin(n\omega t - \theta_n)$$

where $\theta_n = \tan^{-1} \left(\frac{n\omega L}{R} \right)$



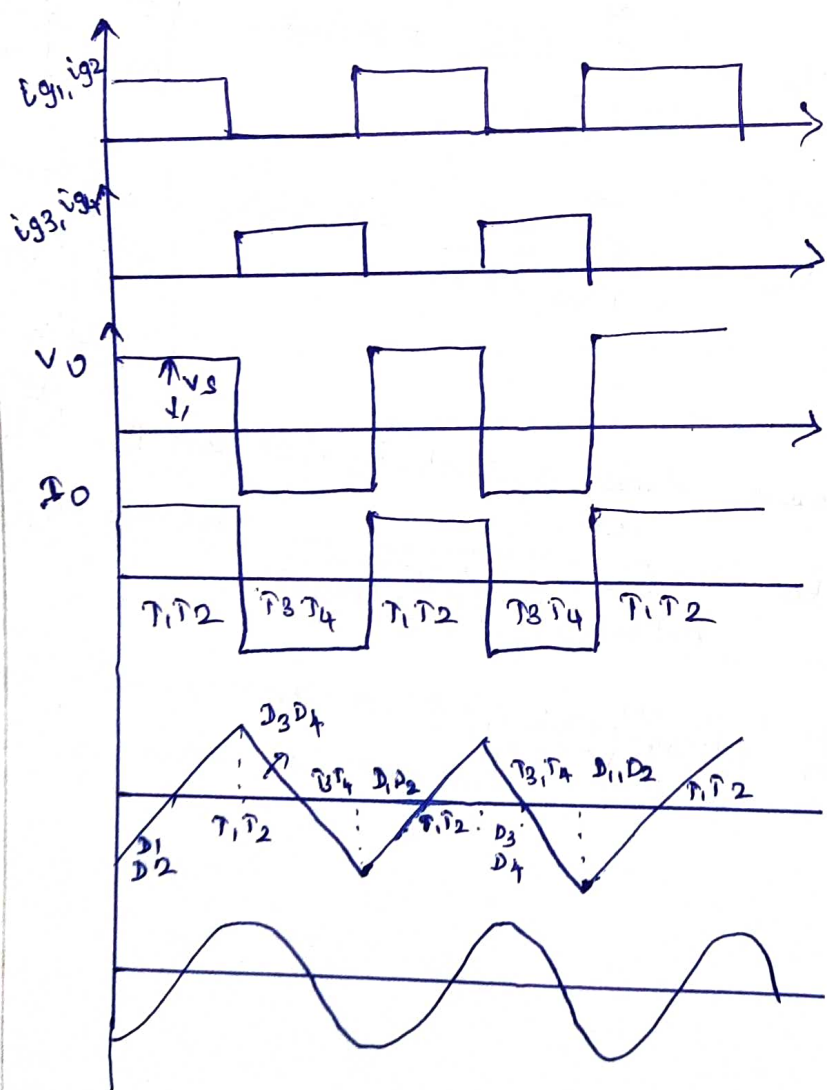
(iii) Distortion factor

voltage control of single phase inverters using various PWM techniques :-

To control the output voltage of inverters it is necessary 1) to cope with the variations of dc input voltage 2) regulate voltage of inverters 3) satisfy the constant volts and frequency control requirements.

The common used techniques are :

- i) single pulse width modulation
- ii) Multiple pulse width modulation
- iii) sinusoidal pulse width modulation.
- iv) Modified sinusoidal pulse width modulation
- v) phase displacement control.



RL load.

load RLC over damped.

For RC circuit,

$$V_s = Ri_0 + \frac{1}{c} \int i_0 dt.$$

$$V_s = R \frac{dq}{dt} + \frac{1}{c} \int \frac{dq}{dt} dt$$

$$V_s = R \frac{dq}{dt} + q/c.$$

$$= R \frac{dq}{dt} + q/c.$$

Taking Laplace transform,

$$R [sQ(s) - q(0)] + \frac{Q(s)}{c} = \frac{V_s}{s}$$

$$Q(s) = \frac{V_s}{R} \frac{1}{s(s + 1/RC)}$$

Taking inverse Laplace transform,

$$q(t) = CV_s (1 - e^{-t/RC}).$$

$$V_c(t) = \frac{q(t)}{c} = V_s (1 - e^{-t/RC}).$$

$$i_0(t) = c \frac{dV_c}{dt} = \frac{V_s}{R} e^{-t/RC}$$

at $t = T/2$,

$$V_c(T/2) = V_s (1 - e^{-\frac{T}{2RC}}).$$

$0 \leq t' \leq T/2$,

$$-V_s = Ri_0 + \frac{1}{c} \int i_0 dt'.$$

$$R \frac{dq}{dt} + q/c = -V_s.$$

$$R [sQ(s) - CV_c(T/2)] + \frac{Q(s)}{c} = -\frac{V_s}{s}$$

$$R S Q(s) - R C V_c (T/2) + \frac{Q(s)}{s} = -\frac{V_s}{s}$$

$$Q(s) \left[R s + \frac{1}{C} \right] = -\frac{V_s}{s} + R C V_c (T/2)$$

$$Q(s) = \frac{-V_s}{s} \left[\frac{C}{C R s + 1} \right] + R C V_c (T/2) \times \frac{C}{R C s + 1}$$

$$= \frac{-C V_s}{s(1 + R C s)} + \frac{C V_c (T/2) \cdot R C}{(s + 1/RC) \times R C}$$

$$= \frac{-C V_s}{s(1 + R C s)} + \frac{C V_c (T/2)}{\left(s + \frac{1}{RC} \right)}$$

$$V_c(t') = -V_s + V_s \left[2 - e^{-\frac{t'}{RC}} \right] e^{-\frac{t'}{RC}}$$

$$i_o(t') = \frac{C \cdot dV_c(t')}{dt} = \frac{-V_s}{R} \left(2 - e^{-\frac{t'}{RC}} \right) \cdot e^{-\frac{t'}{RC}}$$

$$\left(\frac{T}{3RC} \right) \quad t' = T \Rightarrow t = 0$$

$$V_c(t) = (2 - 1) V_s = V_s$$

$$i_o(t) = \left[\frac{1}{s} + \frac{1}{s + 1/RC} \right] \cdot \frac{-V_s}{R}$$

$$V_c = -\frac{V_s}{R} \left[\frac{1}{s} + \frac{1}{s + 1/RC} \right]$$

$$\frac{V_c}{s} = \frac{(-V_s)}{s} \left[\frac{1}{s} + \frac{1}{s + 1/RC} \right]$$

under steady state conditions,

$$\text{at } t=0; i_0(0) = -I_0$$

$$\frac{V_s}{s} = I(s) [R + Ls] + L \cdot I_0$$

$$i_0(t) = \frac{V_s}{R} (1 - e^{-R/Lt}) - I_0 \cdot e^{-R/Lt}$$

$$\text{at } t = T/2, i_0(t) = I_0$$

$$I_0 = \frac{V_s}{R} (1 - e^{-\frac{RT}{2L}}) - I_0 e^{-\frac{RT}{2L}}$$

$$I_0 = \frac{V_s}{R} \frac{1 - e^{-\frac{RT}{2L}}}{1 + e^{-\frac{RT}{2L}}}$$

$$i_0(t) = \frac{V_s}{R} [1 - e^{-\frac{R}{L}t}] - \frac{V_s}{R} \frac{1 - e^{-\frac{RT}{2L}}}{1 + e^{-\frac{RT}{2L}}} e^{-\frac{R}{L}t}$$

$$\text{at } t = T/2; i_0(T/2) = -I_0$$

$$-\frac{V_s}{R} = I(s) [R + Ls] - L I_0$$

$$= -\frac{V_s}{R} [1 - e^{-R/Lt'}] + I_0 e^{-\frac{R}{L}t'}$$

$$I(s) = \frac{-V_s}{s(R+Ls)} + \frac{L \cdot i_0(T/2)}{R+Ls}$$

$$i_0(t') = -\frac{V_s}{R} (1 - e^{-\frac{R}{L}t'}) + i_0(T/2) e^{-\frac{R}{L}t'}$$

$$= -\frac{V_s}{R} (1 - e^{-\frac{R}{L}t'}) + \frac{V_s}{R} (1 - e^{-\frac{RT}{2L}}) e^{-\frac{R}{L}t'}$$

$$= -\frac{V_s}{R} + \frac{V_s}{R} (2 e^{-\frac{R}{L}t'}) + \frac{V_s}{R} e^{-\frac{RT}{2L}} e^{-\frac{R}{L}t'}$$

$$i_o(t') = -\frac{V_s}{R} + \frac{V_s}{R} \left[2 - e^{-\frac{R}{2L}t'} \right] e^{-\frac{R}{L}t'}$$

$$0 \leq t' \leq T/2$$

under steady state conditions,

$$\text{at } t=0; i_o(0) = -I_0$$

$$\frac{V_s}{s} = I(s) [R + Ls] + LI_0$$

$$i_o(t) = \frac{V_s}{R} (1 - e^{-\frac{R}{L}t}) - I_0 e^{-R/Lt}$$

$$\text{at } t = T/2, i_o(t) = I_0$$

$$I_0 = \frac{V_s}{R} (1 - e^{-\frac{RT}{2L}}) - I_0 e^{-\frac{RT}{2L}}$$

$$I_0 = \frac{V_s}{R} \frac{1 - e^{-\frac{RT}{2L}}}{1 + e^{-\frac{RT}{2L}}}$$

$$i_o(t) = \frac{V_s}{R} \left[1 - e^{-\frac{R}{L}t} \right] - \frac{V_s}{R} \frac{1 - e^{-\frac{RT}{2L}}}{1 + e^{-\frac{RT}{2L}}} e^{-\frac{R}{L}t}$$

$$\text{at } t = T/2, i_o(T/2) = I_0$$

$$\frac{V_s}{R} = I(s) [R + Ls] - LI_0$$

$$= -\frac{V_s}{R} \left[1 - e^{-\frac{R}{L}t'} \right] + \frac{V_s}{R} \frac{1 - e^{-\frac{RT}{2L}}}{1 + e^{-\frac{RT}{2L}}}$$

$$e^{-\frac{R}{L}t'}$$

If $T/2 - t_1 > t_2 \rightarrow$ natural commutation.

RL load :-

$$V_s = Ri_0 + L \cdot \frac{di_0}{dt} \quad 0 \leq t \leq T/2.$$

T, T,

$$\frac{V_s}{s} = RI(s) + L [sI(s) - I(0)] = RI(s) + LsI(s) = I(s) [R + Ls]$$

$$I(s) = \frac{V_s}{s(R+Ls)} = \frac{V_s}{sL} \cdot \frac{1}{\left(\frac{R}{L} + s\right)}$$

$$i(t) = \frac{V_s}{R} (1 - e^{-R/L t})$$

at $t = T/2$

$$i_0(T/2) = \frac{V_s}{R} (1 - e^{-R/L \cdot T/2})$$

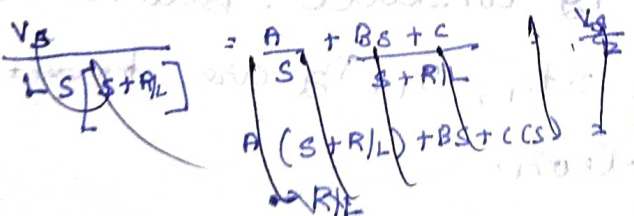
$T/2 \leq t \leq T$

$$-V_s = Ri_0 + L \cdot \frac{di_0}{dt} \quad T/2 \leq t \leq T.$$

$$I(s) = \frac{-V_s}{s(R+Ls)} + \frac{L \cdot i_0(T/2)}{R+Ls}$$

$$i_0(t) = \frac{-V_s}{R} (1 - e^{-R/L t'}) + i_0(T/2) e^{-R/L t'}$$

$$i_0(t') = \frac{-V_s}{R} + \frac{V_s}{R} \left[2 - e^{-\frac{R^2}{2L}} \right] e^{-R/L t'}$$



$$-V_s = Ri_0 + L \cdot \frac{di_0}{dt} ; T/2 \leq t \leq T.$$

$$\frac{-V_s}{s} = RI(s) + L [sI(s) - I(0)].$$

$$-V_s/s = RI(s) + L \cdot sI(s) - L \cdot I(0).$$

$$-V_s/s = I(s) [R + Ls] - L I(0).$$

$$-V_s/s + L I(0) = I(s) (R + Ls).$$

$$I(s) = \frac{-V_s}{s(R+Ls)} + \frac{L \cdot I(0)}{R+Ls}$$

Partial S = -R/L
 $V_s = \frac{V_s}{R} [0] + B(-R/L)$
 $B = \frac{-V_s \cdot L}{R}$
 $A = \frac{V_s}{R}$
 $V_s = A(R+Ls) + B \cdot s$
 $V_s = 0, V_s = A \cdot R, A = V_s/R$
 $\frac{V_s}{s(R+Ls)} = \frac{A}{s} + \frac{B}{R+Ls}$

$$0 \leq t \leq T/2$$

$$V_s = Ri_0 + L \frac{di_0}{dt} + \frac{1}{C} \int i_0 dt + V_{c1}$$

$V_{c1} \rightarrow$ voltage across the capacitor at $t=0$.

$$T/2 \leq t \leq T \text{ (or)} 0 \leq t' \leq T/2, t' = t - T/2$$

$$-V_s = Ri_0 + L \frac{di_0}{dt} + \frac{1}{C} \int i_0 dt' + V_{c2}$$

$V_{c2} \rightarrow$ voltage across capacitor at $t'=0$.

Differentiating both the equations,

$$\frac{d^2 i_0}{dt^2} + \frac{R}{L} \frac{di_0}{dt} + \frac{1}{LC} i_0 = 0$$

$$\frac{d^2 i_0}{dt'^2} + \frac{R}{L} \frac{di_0}{dt'} + \frac{1}{LC} i_0 = 0$$

Solving these two equations, i_0 will be obtained

In RL and RLC over damped:

At $t=0$, T_1, T_2 are triggered. But the current direction cannot be change immediately. D_1, D_2 starts conduct.

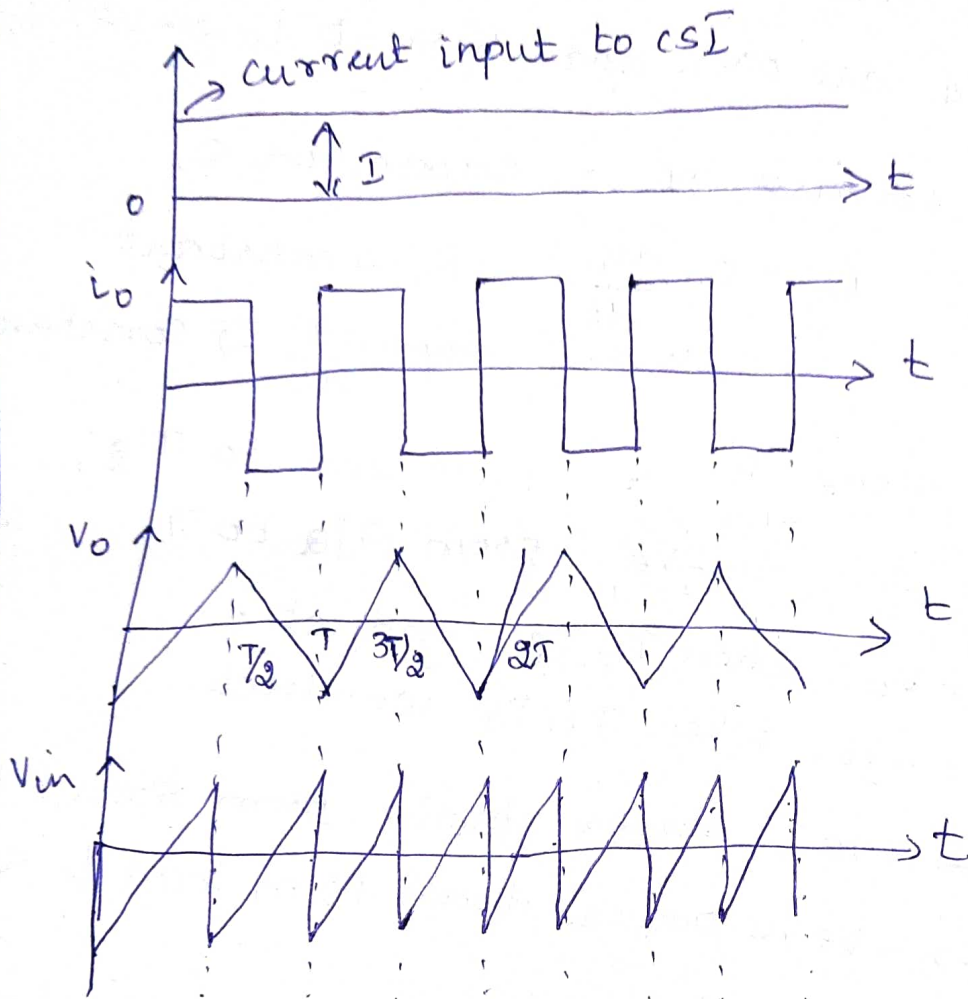
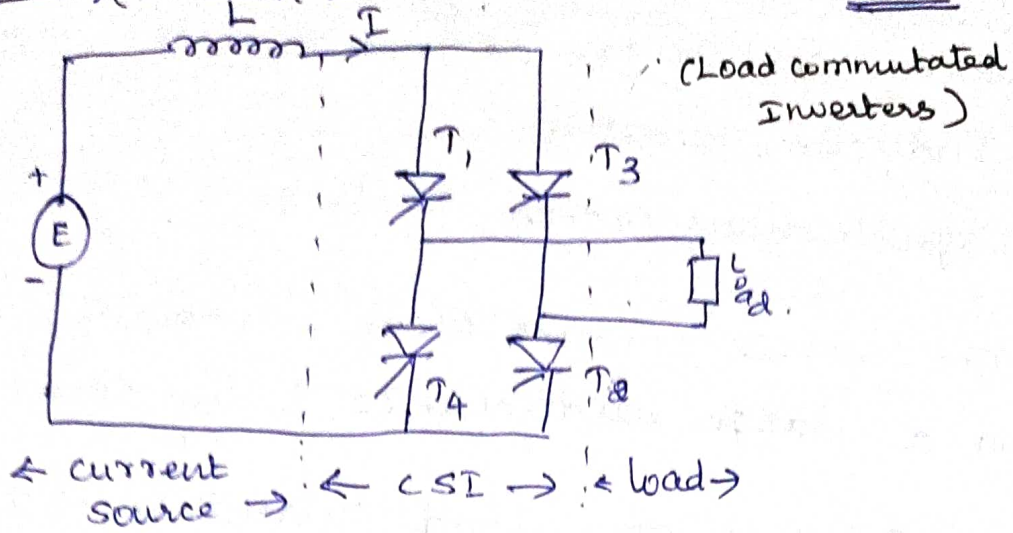
at $t=T/2$, T_1, T_2 are force commutated.

RLC underdamped load :-

after $t=0$, T_1, T_2 conduct. But due to the load nature, at $t=t_1$, T_1, T_2 are ~~used~~ ~~into~~ ~~action~~ D_1, D_2 into action.

Current Source Inverters :-

UNIT - IV



- ⇒ current source inverter (CSI), input current is constant but adjustable.
- ⇒ The amplitude of current from CSI is independent of the load.
- ⇒ Does not require any feedback diodes.

applications :-

- i) speed control of ac motors
- ii) Induction heating
- iii) synchronous motor starting.

⇒ The source consists of a voltage source E and a large inductance L in series with it.

⇒ T_1, T_2 are ON, load current $i_o \rightarrow +ve, i_o = I.$

T_3, T_4 are ON, load current $i_o \rightarrow -ve, i_o = -I.$

⇒ Load consists of a capacitor $C,$

$$i_o = C \cdot \frac{dv_o}{dt} \quad i_o \rightarrow \text{constant}$$

slope $\frac{dv_o}{dt} \rightarrow \text{constant}.$

⇒ This slope is +ve, from zero to $T/2,$
-ve from $T/2$ to $T.$

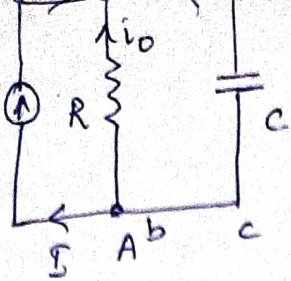
$V_{in} = V_o,$ when T_1, T_2 conduct.

$V_{in} = -V_o,$ when T_3, T_4 conduct.

⇒ $V_{in} \rightarrow +ve,$ power flows from source to load.

$V_{in} \rightarrow -ve,$ power flows from load to source.

⇒ CSI may be load or force commutated.



$$i_o + i_c + I = 0$$

$$i_c = -I - i_o$$

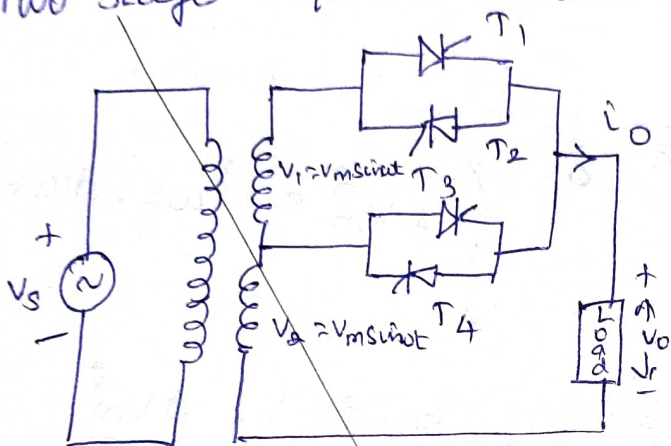
$$\text{At } t = T/2, i_o = +I_1$$

$$i_c = -I - I_1 = -(I + I_1)$$

$$\text{At } t = T; i_o = -I_1$$

$$i_c = -I + I_1 = -(I - I_1)$$

Two stage sequence control of voltage controllers :-



$$V_s = V_m \sin \omega t$$

$$V_1 = V_2 = V_m \sin \omega t$$

Sum of two secondary voltages is $2V_m \sin \omega t$.

Advantage :-

Reduction of harmonics in the load and supply currents.

Resistance Load

when both pairs T_1, T_2 & T_3, T_4 are in operation, firing angle for T_3, T_4 is always zero, & for pair T_1, T_2 is varied from 180° to zero, for obtaining output voltage from V to $2V$.

\Rightarrow SCR T_1 is triggered, at $\omega t = \alpha$,
 V_1 reverse biases T_3 , it is turned off.

\Rightarrow T_1 begins to conduction, output voltage jumps from V_2 to $(V_1 + V_2)$.

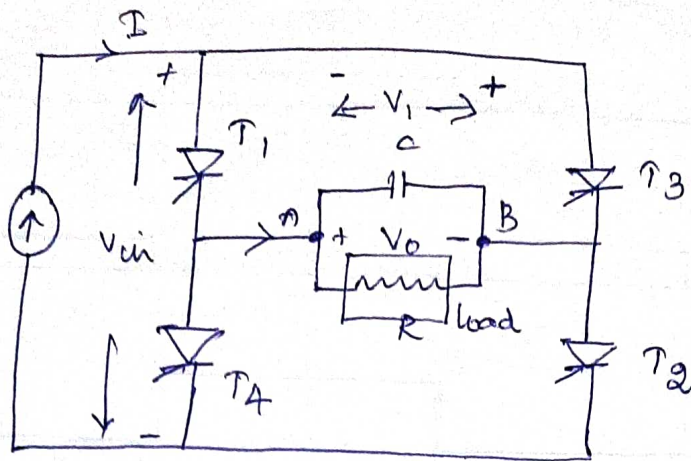
\Rightarrow T_4 is triggered \rightarrow output voltage follows $V_m \sin \alpha$

$$V_{or} = \left[\frac{1}{\pi} \int_0^\alpha V_m \sin \omega t \, d(\omega t) + \int_\alpha^\pi A V_m \sin \omega t \cdot d(\omega t) \right]^{1/2}$$

$$= \frac{V_m}{2\pi} \left(\alpha - \frac{\sin 2\alpha}{2} \right) + \frac{2V_m}{\pi}$$

$$\left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right)^{1/2}$$

1 ϕ capacitor-commutated CSI with R-load :-



→ capacitor C in parallel with the load is used for storing the charge for commutating the SCRs.

→ T_1, T_2 together gated by i_{g1}, i_{g2} .

T_3, T_4 together gated by i_{g3}, i_{g4} .

Before $t=0$, $V_c = -V_i$, Left plate $-ve$,
Right plate $+ve$.

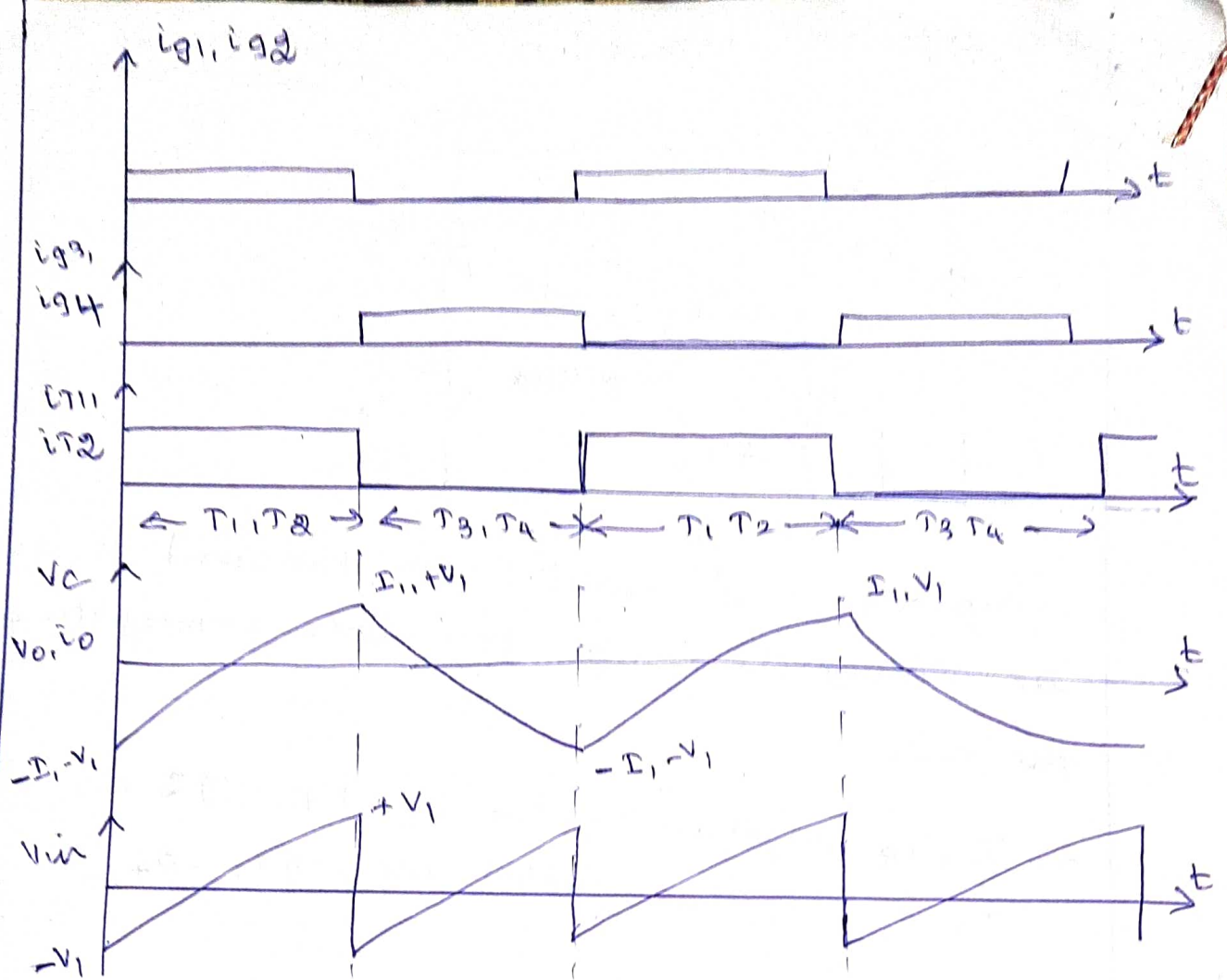
→ when T_1, T_2 are gated at $t=0$, V_c reverse biases conducting thyristors T_3, T_4 .

→ source current I flows through T_1 , parallel combination of R and C through T_2 .

→ From zero to $T/2$, $i_{T1} = i_{T2} = I$.

→ when T_3, T_4 are gated at $t = T/2$, $V_c = V_i$,
 T_1, T_2 reverse biases.

→ source current I flows through T_3 , parallel combination of R and C , $T/2$ to T .



At $t = 0$, capacitor charged with $V_c = -V_1$.

load current $i_o = \frac{-V_1}{R} = -I_1$.

$t = 0$ to $T/2$; capacitor charges from $-V_1$ to V_1 .

$t = T/2$; $i_o = \frac{V_0}{R} = \frac{V_1}{R} = I_1$.

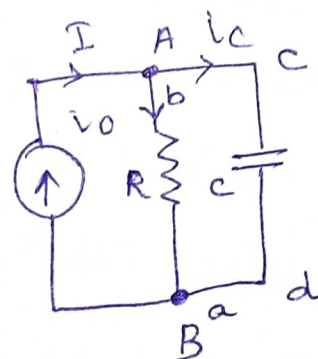
KCL at node A,

$$i_o + i_c = I$$

$$i_c = I - i_o$$

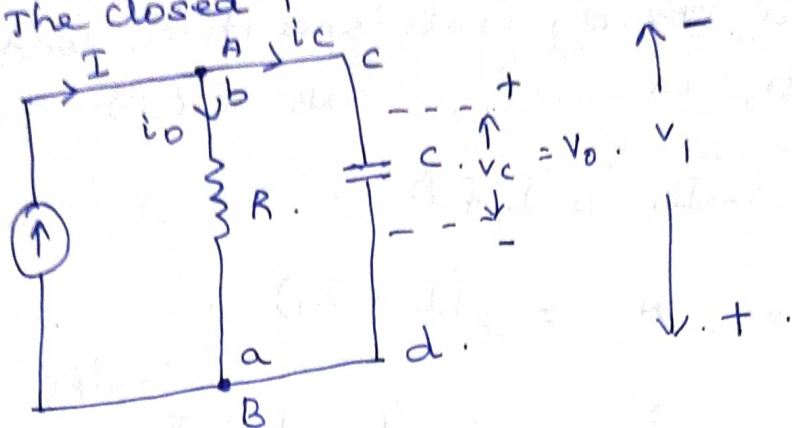
At $t = 0$, $i_o = -I_1$, $i_c = I + I_1$.

before $T/2$, $i_o = I_1$, $i_c = I - I_1$.



Analysis :-

The capacitor is initially charged to a voltage $-V_1$. The closed path abcda we get,



$$R i_0 - \frac{1}{C} \int (I - i_0) dt + V_1 = 0 \quad \text{--- (1)}$$

differentiate with respect to time,

$$R \frac{di_0}{dt} - \frac{I}{C} + \frac{i_0}{C} = 0$$

$$R \frac{di_0}{dt} + \frac{i_0}{C} = \frac{I}{C}$$

$$\left[R p + \frac{1}{C} \right] i_0 = \frac{I}{C} \quad \text{--- (2)}$$

complementary solution function of the solution is $\left[R p + \frac{1}{C} \right] I_{cp} = 0$.

$$R p = -\frac{1}{C}$$

$$p = -\frac{1}{RC}$$

$$I_{cp} = A \cdot e^{-t/RC}$$

For particular integral, put $p=0$,

$$\frac{i_0}{C} = \frac{I}{C} \quad \text{on } i_0 = I$$

complete solution for load current i_0 ,

$$i_0 = P \cdot I + C \cdot F.$$

$$i_0 = I + A \cdot e^{-t/RC} \quad \text{--- (3)}$$

under steady state operation, load current at $t = 0$, $i_0 = -I_1$, sub in (3).

$$-I_1 = I + A.$$

$$A = -(I + I_1).$$

$$i_0 = I - (I + I_1) \cdot e^{-t/RC}$$

$$i_0 = I - I \cdot e^{-t/RC} - I_1 \cdot e^{-t/RC}$$

$$i_0 = I [1 - e^{-t/RC}] - I_1 \cdot e^{-t/RC}$$

$$0 < t < T/2. \quad \text{--- (4)}$$

at $t = T/2$, $i_0 = I_1$. sub in (4),

$$I_1 = I [1 - e^{-\frac{T}{2RC}}] - I_1 \cdot e^{-\frac{T}{2RC}}$$

$$\cancel{I_1} + I_1 \cdot e^{-\frac{T}{2RC}} = I [1 - e^{-\frac{T}{2RC}}]$$

$$I_1 + I_1 \cdot e^{-\frac{T}{2RC}} = I [1 - e^{-\frac{T}{2RC}}]$$

$$I_1 [1 + e^{-\frac{T}{2RC}}] = I [1 - e^{-\frac{T}{2RC}}] \quad \text{--- (5)}$$

$$I_1 = I \left[\frac{1 - e^{-\frac{T}{2RC}}}{1 + e^{-\frac{T}{2RC}}} \right]$$

$$= I \quad \text{if } \frac{T}{2RC} \gg 1, \\ T \gg RC.$$

sub eqn ⑤ in eqn ④,

$$i_o = I \left[1 - e^{-t/RC} \right] - I \left[\frac{1 - e^{-\frac{t}{2RC}}}{1 + e^{-\frac{t}{2RC}}} \right] e^{-t/RC}$$

output voltage V_o (or) capacitor voltage V_c is,

$$V_o = V_c = R i_o = RI \left[1 - 2 \frac{e^{-t/RC}}{1 + e^{-\frac{t}{2RC}}} \right]$$

$$i_o = \frac{I \left[1 - e^{-t/RC} \right] \left[1 + e^{-\frac{t}{2RC}} \right] - I \left[1 - e^{-\frac{t}{2RC}} \right] e^{-t/RC}}{1 + e^{-\frac{t}{2RC}}}$$

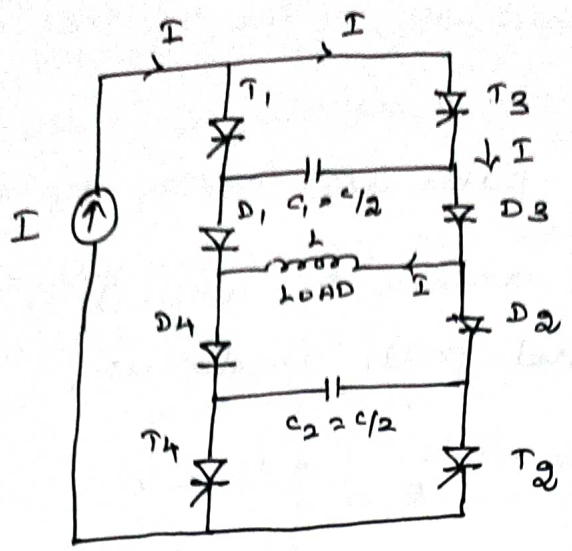
$$i_o = I \left[1 - 2 \frac{e^{-t/RC}}{1 + e^{-\frac{t}{2RC}}} \right]$$

turn off time t_c , provided by the circuit to each SCR is obtained when $t = t_c$, $V_o = V_c = I_o R = 0$.

$$V_o = V_c = R i_o = RI \left[1 - 2 \frac{e^{-t_c/RC}}{1 + e^{-\frac{t_c}{2RC}}} \right] = 0$$

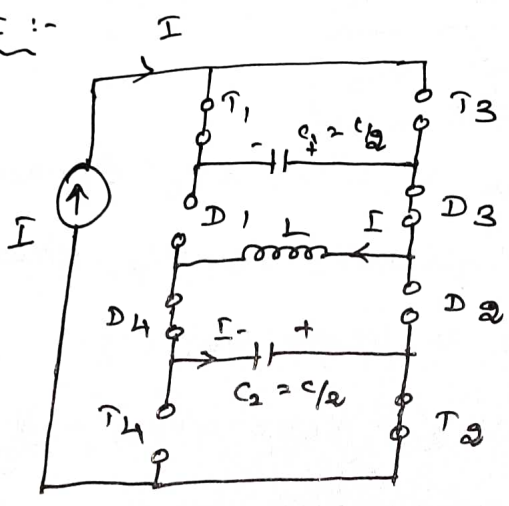
$$t_c = RC \ln \left[\frac{2}{1 + \exp(-\frac{t_c}{2RC})} \right]$$

Single phase Auto-sequential Commutated Inverter :-



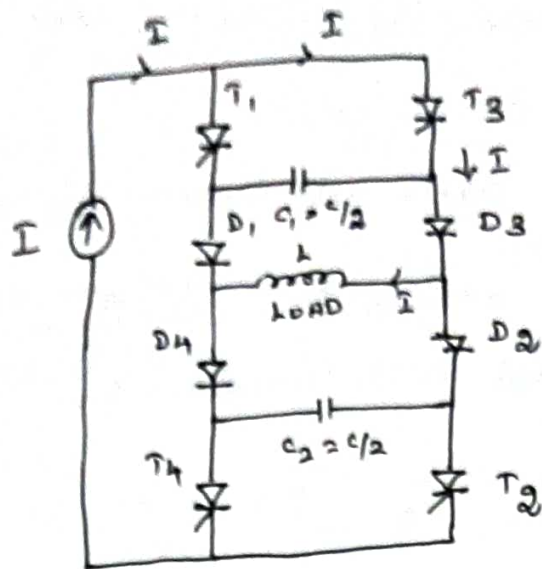
- => Thyristor pairs T_1, T_2, T_3, T_4 are alternatively switched to obtain a nearly square wave load current.
- => Two commutating capacitors, one C_1 in the upper half and the other C_2 in the lower half are connected.
- => Diodes D_1 to D_4 are connected in series to prevent the commutation capacitors from discharging into the load.

MODE I :-



- before $t = 0$, assume that T_3, T_4 are conducting and a steady current I flows through the path T_3, D_3, L, D_4, T_4 and source I .
- => commutating capacitors are assumed to be initially charged equally with polarity $V_{C1} = V_{C2} = -V_{CD}$.

Single phase Auto-sequential Commutated Inverter :-

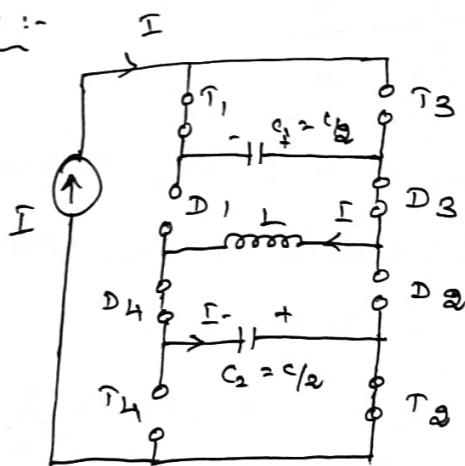


\Rightarrow Thyristor pairs T_1, T_3, T_3, T_4 are alternatively switched to obtain a nearly square wave load current.

\Rightarrow Two commutating capacitors, one C_1 in the upper half and the other C_2 in the lower half are connected.

\Rightarrow Diodes D_1 to D_4 are connected in series to prevent the commutation capacitors from discharging into the load.

MODE I :-



before $t=0$, assume that T_3, T_4 are conducting and a steady current I flows through the path T_3, D_3, L, D_4, T_4 and source I .

\Rightarrow commutating capacitors are assumed to be initially charged equally with polarity $V_{C1} = V_{C2} = -V_{CO}$.

At $t = 0$, T_1, T_2 are gated.

T_3, T_4 are turned off by the reverse capacitor voltages.

T_1, T_2 conducts, path are $T_1, C_1, D_3, L, D_4, C_2, T_2$.

The voltage V_{D1} across D_1 , when it is forward biased, by closed path abcda as,

$$V_{D1} + V_{C0} - \frac{1}{C/2} \int I dt = 0.$$

voltage across L is zero, because of constant current I .

$$V_{D1} = -V_{C0} + \frac{q}{c} \int I dt.$$

capacitor charges, voltages V_{D1} across D_1 rises linearly.

at $t = t_1$, $V_{D1} = 0$,

$$0 = -V_{C0} + \frac{q}{c} \int I t_1.$$

$$t_1 = \frac{c}{2I} V_{C0}.$$

capacitor voltage $V_{C1} = V_{C2} = V_c$ appears as reverse voltage across thyristors T_3, T_4 when T_1, T_2 gated.

The value of V_c is given as,

$$V_{C1} = V_{C2} = V_c = -V_{C0} + \frac{q}{c} \int I dt.$$

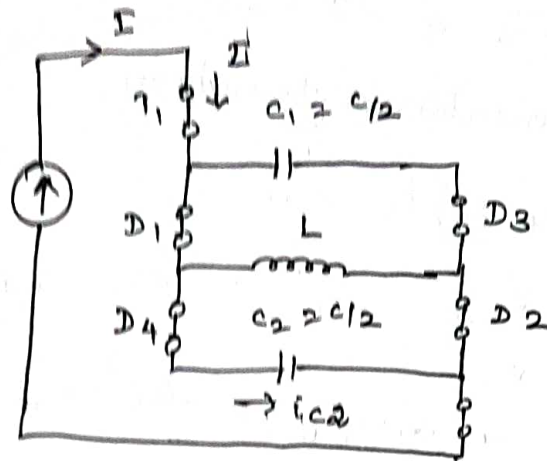
$$\text{at time } t_1, V_{C1} = V_{C2} = V_c(t_1) = -V_{C0} + \frac{q}{c} I t_1.$$

$$\text{sub } t_1 = \frac{c}{2I} V_{C0}.$$

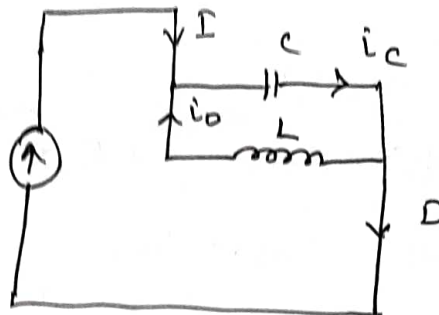
$$V_{C1} = V_{C2} = V_c(t_1) = -V_{C0} + \frac{qI}{c} \left(\frac{c}{2I} V_{C0} \right) = 0.$$

Diodes D_3, D_4 are already conducting but at $t = t_1$, diodes D_1, D_2 get forward biased and start conducting.

2) at the end of t_1 , all four diodes D_1, D_2, D_3, D_4 conduct.



MODE II.



Equivalent circuit

In equivalent circuit, KCL gives

$$I + i_o = i_c (= i_{c1} + i_{c2}).$$

$$i_{c1} = i_{c2}, \quad i_{c1} = i_{c2} = \frac{i_c}{2}.$$

$$\text{KVL gives } L \cdot \frac{di_o}{dt} + \frac{1}{C} \int i_c \cdot dt = 0.$$

$$L \cdot \frac{di_o}{dt} + \frac{1}{C} \int (I + i_o) dt = 0.$$

$$L \cdot \frac{d^2 i_o}{dt^2} + \frac{i_o}{C} = -\frac{I}{C} \dots \dots \textcircled{1}$$

$$\frac{d^2 i_o}{dt^2} + \frac{i_o}{LC} = -\frac{I}{LC}$$

Solving the equation,

$$i_0 = I, \quad \frac{di_0}{dt} = 0.$$

In eqn (1), for particular integral,

$$\frac{\dot{i}_{os}}{c} = -\frac{I}{c}.$$

$$\dot{i}_{os} = -I.$$

for complementary function,

$$(Lp^2 + \frac{1}{c}) i_0 = 0.$$

$$Lp^2 + \frac{1}{c} = 0.$$

$$p^2 = -\frac{1}{Lc} = -\omega_0^2 = j^2 \omega_0^2.$$

$$p = \pm j\omega_0.$$

$$\omega_0 = \frac{1}{\sqrt{Lc}}$$

$$i_0(t) = A \cdot e^{j\omega_0 t} + B e^{-j\omega_0 t}.$$

$$\begin{aligned} \dot{i}_0(t) &= \dot{i}_{os} + \dot{i}_{ot} \\ &= -I + A e^{j\omega_0 t} - j\omega_0 B e^{-j\omega_0 t} \end{aligned} \quad \dots (1)$$

$$t = 0, \quad \dot{i}_0 = I.$$

$$I = -I + A + B.$$

$$\boxed{A + B = 2I} \quad \dots (2).$$

at $t = 0$, $\frac{di_0}{dt} = 0$, from (1),

$$\frac{di_0}{dt} = j\omega_0 A e^{j\omega_0 t} - j\omega_0 B e^{-j\omega_0 t} = 0.$$

$$j\omega_0 (A - B) = 0.$$

$$(A - B) = 0. \quad \dots (3).$$

i_0 consists of two components:

(i) steady state component

(ii) Transient component.

i_0 is a transient component $i_0 = A \cos \omega_0 t + B \sin \omega_0 t$.

steady state component:

$$L \frac{di_0}{dt} + \frac{1}{C} \int (i_0 + I) dt = 0.$$

$$\frac{di_0}{dt} = 0.$$

$$\frac{i_0 + I}{C} = 0.$$

$$i_0 = -I.$$

Total current $i_0 = -I + A \cos \omega_0 t + B \sin \omega_0 t$.

at $t = 0$, $i_0 = I$

$$i_0 = -I + A.$$

$$A = I + I = 2I.$$

$$i_0 = -I + 2I \cos \omega_0 t.$$

$$i_0 = I [2 \cos \omega_0 t - 1]$$

$$i_c = i_0 + I$$

$$i_c = I [2 \cos \omega_0 t - 1] + I.$$

$$i_c = I [2 \cos \omega_0 t].$$

Voltage across the capacitor,

$$V_c = \frac{1}{C} \int i_c dt.$$

$$= \frac{1}{C} \int I (2 \cos \omega_0 t) dt.$$

$$= \frac{I}{C} \frac{2 \sin \omega_0 t}{\omega_0} = \frac{2I}{\omega_0 C} \sin \omega_0 t.$$

$i_0 = I [2 \cos \omega_0 t - 1]$

$$i_{c1} = i_{ca} = \frac{i_c}{2} = \frac{2I \cos \omega_0 t}{2} = I \cos \omega_0 t$$

$$\begin{aligned} i_{D3} &= I - i_{c1} \\ &= I - I \cos \omega_0 t \\ &= I [1 - \cos \omega_0 t] \end{aligned}$$

A time t_2 must elapse for the current i_{c1} to become zero. This time t_2 can be obtained by equating i_{c1} to zero.

$$\begin{aligned} i_{c1} &= I \cos \omega_0 t_2 = 0 \\ \cos \omega_0 t_2 &= \cos \pi/2 \end{aligned}$$

$$t_2 = \frac{\pi}{2\omega_0}$$

Total commutation interval t_c is

$$t_c = t_1 + t_2 = \frac{C}{2I} V_{CO} + \frac{\pi}{2\omega_0}$$

$$\begin{aligned} t_1 &= \frac{C}{2I} V_{CO} = \frac{C}{2I} \times \frac{2I}{\omega_0 C} \\ &= \frac{1}{\omega_0} = \sqrt{LC} \end{aligned}$$

$$t_c = \sqrt{LC} + \frac{\pi}{2} \sqrt{LC} = \sqrt{LC} \left[1 + \frac{\pi}{2} \right]$$

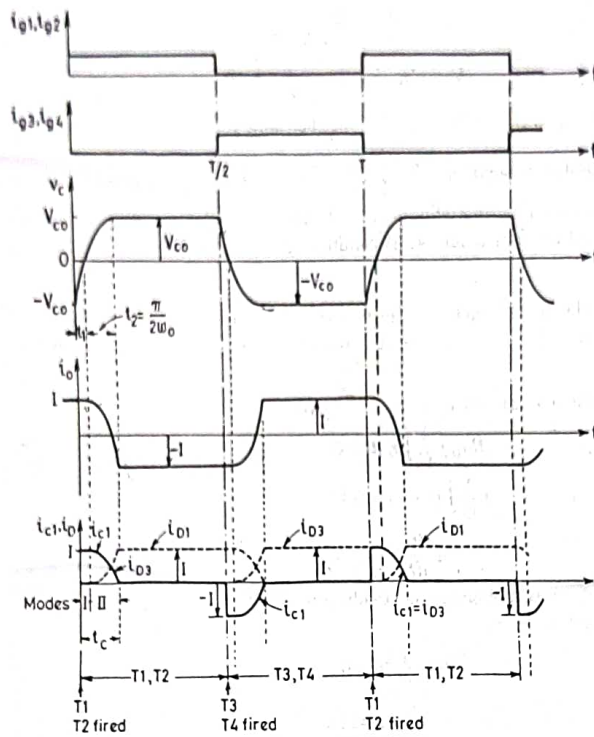
At the end of total commutation interval $(t_1 + t_2)$, the steady input current I flows through T_1, D_1, L, D_2, T_2 . This current continues to flow till the next commutation process.

FROM EQN (A) & (B), $A = B = I$,

FROM (1) $i_c(t) = -I + 2I \left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \right]$.

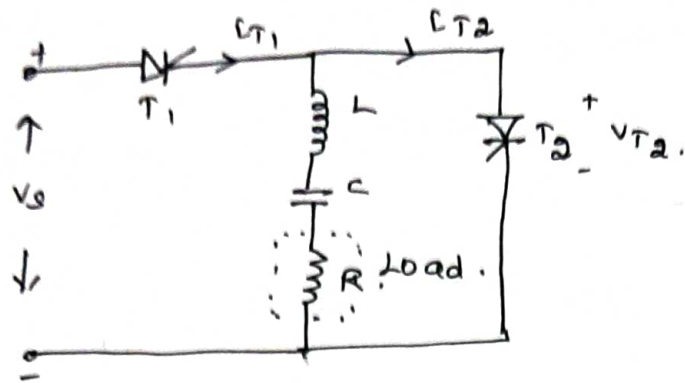
$i_c(t) = I [2 \cos \omega_0 t - 1]$.

$v_c(t) = \frac{2I}{\omega_0 C} \sin \omega_0 t$.



SERIES INVERTER :-

Inverters in which commutating components are permanently connected in series with the load are called series inverters. The series circuit must be underdamped. series inverter also called load commutated inverters or self commutated inverters.



⇒ It consists of load resistance R , in series with commutating components L and C .

⇒ When T_1 is turned on, T_2 off, i starts building up in the RLC circuit.

⇒ The load current after reaching peak value, decays to zero at a point a .

⇒ at pt a , load current tends to reverse.

⇒ minimum time is given by,

$$t_{q, \min} = \frac{\pi}{\omega} - \frac{\pi}{\omega r} = \frac{1}{2} \left(\frac{1}{f} - \frac{1}{fr} \right)$$

ω → output frequency r/s .

ωr → circuit ringing frequency in r/s .

T_1 → OFF, T_2 → ON, $T_{off} > t_{q, \min}$.

C → discharge, load current builds up in the reverse direction, to some peak negative value and decays to zero.

After this time $T_{off} = cd$ must elapse for T_a to recover. At d , T_1 is again turned on. The process repeats.

Analysis of Basic Series Inverter :-

when T_1 is turned on,

$$Ri + L \frac{di}{dt} + \frac{1}{c} \int i dt = V_s \dots (1)$$

with zero initial conditions, L.T is,

$$I(s) \left[R + Ls + \frac{1}{sc} \right] = \frac{V_s}{s}$$

$$I(s) = \frac{V}{L} \cdot \frac{1}{s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}} \dots (2)$$

root of $s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC} = 0$ are

$$s = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

The circuit is underdamped,

$$\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \text{ must be negative.}$$

$$\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} < 0, R^2 < \frac{4L}{c}$$

$$s = -\frac{R}{2L} \pm j \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$s = -\xi + j\omega_r$$

$$\xi = -R/2L, \omega_r = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$\text{If } \omega_0 = \frac{1}{\sqrt{LC}}, \omega_d = \sqrt{\omega_0^2 - \xi^2}$$

$$\omega_0 = \sqrt{\omega_d^2 + \xi^2}$$

from (a),

$$I(s) = \frac{V_s}{L} \left[\frac{1}{(s+\xi-j\omega_r)(s+\xi+j\omega_r)} \right]$$

Let $\frac{1}{(s+\xi-j\omega_r)(s+\xi+j\omega_r)} = \frac{A}{s+\xi-j\omega_r} + \frac{B}{s+\xi+j\omega_r}$

$$A = \frac{1}{2j\omega_r} ; B = \frac{-1}{2j\omega_r}$$

$$I(s) = \frac{V_s}{L} \cdot \frac{1}{\omega_r} \left[\frac{\omega_r}{(s+\xi)^2 + \omega_r^2} \right]$$

inverse Laplace Transform is,

$$i(t) = \frac{V_s}{\omega_r \cdot L} e^{-\xi t} \sin \omega_r t \dots (3)$$

Resonant frequency $f_r = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{\frac{1}{Lc} - (R/2L)^2}}$ Hz, $f < f_r$.

From eqn (3),

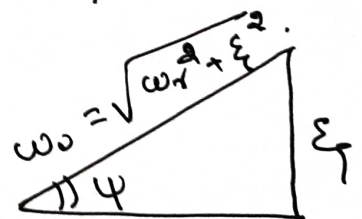
$$V_L = L \cdot \frac{di}{dt} = L \cdot \frac{V_s}{L} \cdot \frac{1}{\omega_r} \left[e^{-\xi t} \cdot \omega_r \cos \omega_r t - \xi \cdot e^{-\xi t} \sin \omega_r t \right]$$

$$V_L = V_s \cdot \frac{\omega_0}{\omega_r} e^{-\xi t} \cdot \cos(\omega_r t + \psi)$$

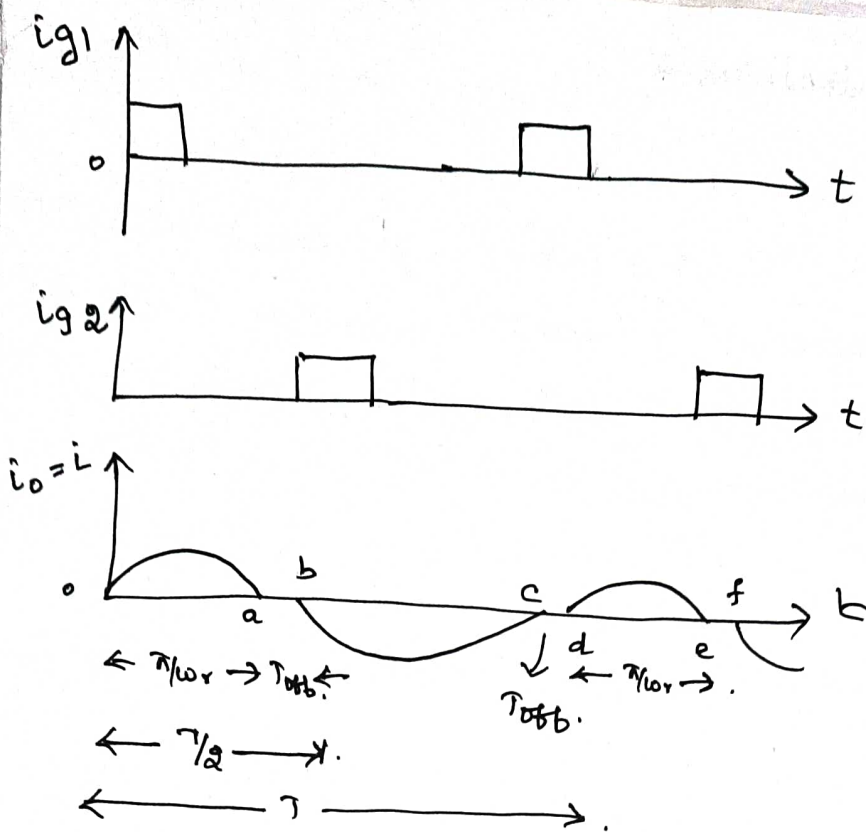
$\omega_0 \rightarrow$ resonant frequency.

$$\omega_0 = \sqrt{\omega_r^2 + \xi^2}$$

$$\psi = \tan^{-1} \left(\frac{\xi}{\omega_r} \right)$$



$$V_C = V_s \left[1 - e^{-\xi t} \frac{\omega_0}{\omega_r} \cdot \cos(\omega_r t - \psi) \right]$$



Load current waveform for basic series inverter.

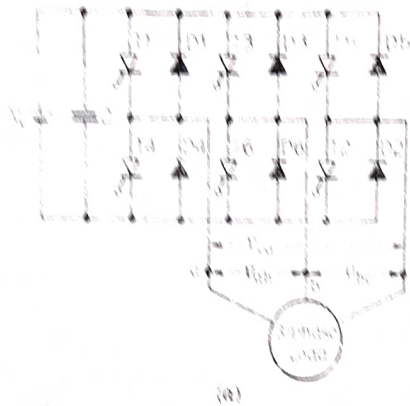
Comparison of VSI & CSI :-

VSI	CSI
1) VSI is fed from a DC voltage source having small impedance.	CSI is fed from DC voltage source of high impedance.
2) input voltage maintained constant	Input is constant but adjustable.
3) output voltage doesnot dependent on the load.	amplitude of current independent of load.
4) VSI requires feedback diodes.	feedback diodes not required.
5) commutation circuit is complicated.	commutation circuit simple.

Inverters.

Single phase and three phase voltage source inverters (both 120° mode and 180° mode) - Voltage & harmonic control - PWM techniques: sinusoidal PWM, modified sinusoidal PWM - multiple PWM - introduction to space vector modulation - current source inverter.

Three phase voltage source inverter (180° mode) :-

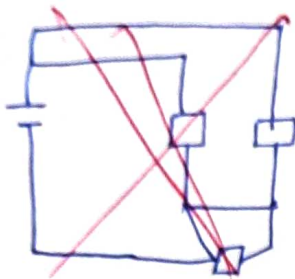
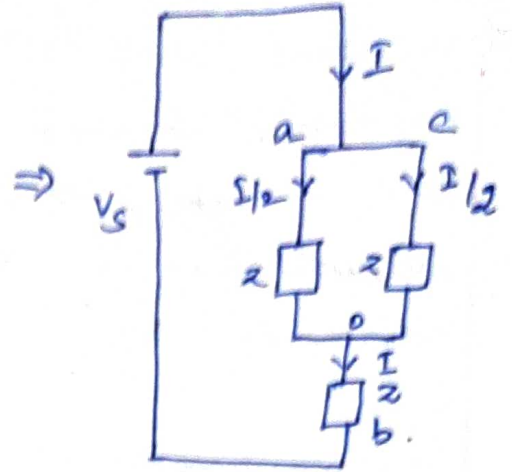
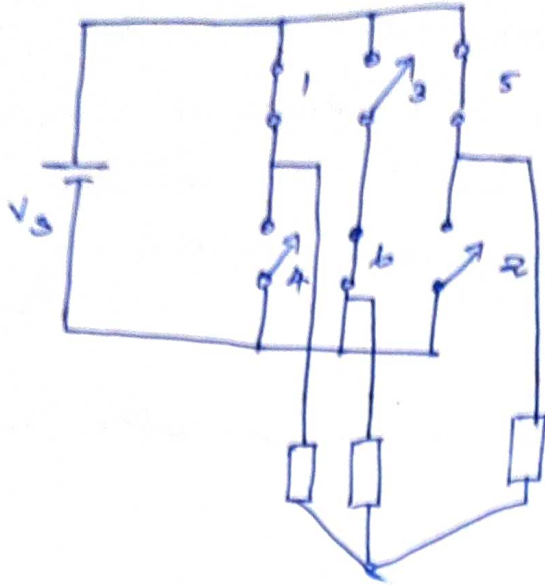


Three phase inverter is a six step bridge inverter. It uses a minimum of 6 thyristors. A step is defined as a change in the firing from one thyristor to the next thyristor in proper sequence. A large capacitor is used to make the input dc voltage constant.

- T_1 triggered at $\omega t = 0^\circ$, and conducts for 180° .
- T_2 triggered at $\omega t = 60^\circ$, and conducts for $60^\circ + 180^\circ = 240^\circ$.
- T_3 triggered at $\omega t = 120^\circ$, conducts for $120^\circ + 180^\circ = 300^\circ$.
- T_4 triggered at $\omega t = 180^\circ$, conducts for $180^\circ + 180^\circ = 360^\circ$.
- T_5 triggered at $\omega t = 240^\circ$, conducts for $240^\circ + 180^\circ = 420^\circ$.
- T_6 triggered at $\omega t = 300^\circ$, conducts for $300^\circ + 180^\circ = 480^\circ$.

Equivalent circuit:

Mode I : $0-60^\circ$, 5, b, 1 are conduct



Total impedance,

$$z \parallel z + z = \frac{z \times z}{z + z} + z$$

$$= \frac{z^2}{2z} + z$$

$$= \frac{3z^2}{2z}$$

$$z_{\text{equ}} = \frac{3z}{2}$$

$$I = \frac{V_s}{z} = \frac{V_s}{\frac{3z}{2}} = \frac{2V_s}{3z}$$

$$V_{ao} = I/2 \times z = \frac{2V_s}{3z} \times \frac{z}{2} = \frac{V_s}{3}$$

$$V_{bo} = -I \times z = -\frac{2V_s}{3z} \times z = -\frac{2V_s}{3}$$

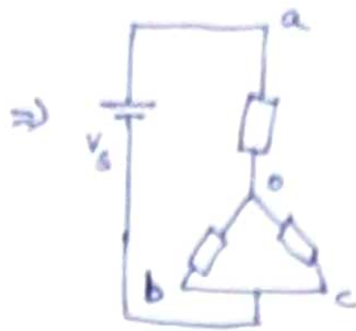
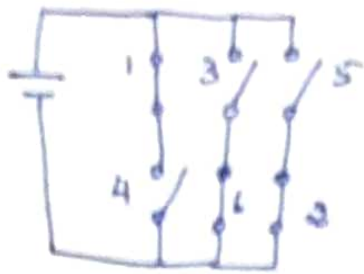
$$V_{co} = I/2 \times z = \frac{2V_s}{3z} \times \frac{z}{2} = \frac{V_s}{3}$$

$$V_{ab} = V_{ao} + V_{ob} = \frac{V_s}{3} + \frac{2V_s}{3} = V_s$$

$$V_{bc} = V_{bo} + V_{oc} = -\frac{2V_s}{3} - \frac{V_s}{3} = -V_s$$

$$V_{ca} = V_{co} + V_{oa} = \frac{V_s}{3} - \frac{V_s}{3} = 0$$

Mode (i) $60^\circ - 180^\circ$, b, l, a are conduct



$$V_{ao} = \frac{2V_s}{3}$$

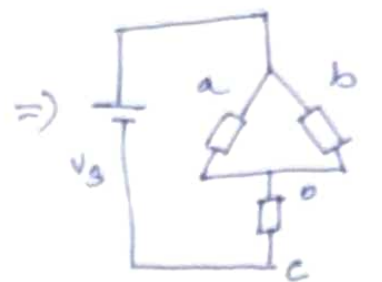
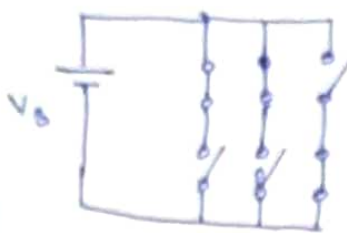
$$V_{ob} = \frac{V_s}{3}$$

$$V_{oc} = \frac{V_s}{3}$$

$$V_{ab} = V_{ao} + V_{ob} = V_s; \quad V_{bc} = V_{bo} + V_{oc} = -\frac{V_s}{3} + \frac{V_s}{3} = 0$$

$$V_{ca} = V_{co} + V_{oa} = -\frac{V_s}{3} - \frac{2V_s}{3} = -V_s$$

Mode (iii) $180^\circ - 180^\circ$, 1, a, 3 are conduct:



$$V_{ao} = \frac{V_s}{3}$$

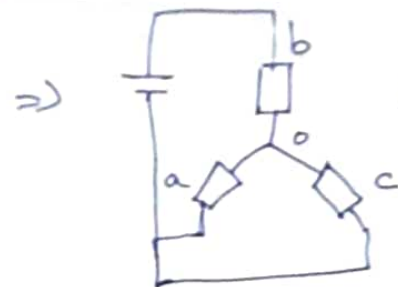
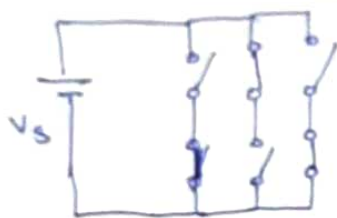
$$V_{bo} = \frac{V_s}{3}$$

$$V_{oc} = \frac{2V_s}{3}$$

$$V_{ab} = V_{ao} + V_{ob} = 0; \quad V_{bc} = V_{bo} + V_{oc} = V_s;$$

$$V_{ca} = V_{co} + V_{oa} = -V_s$$

Mode (iv); $180^\circ - 240^\circ$, 2, 3, 4 conduct.



$$V_{bo} = \frac{2V_s}{3}$$

$$V_{ao} = \frac{V_s}{3}$$

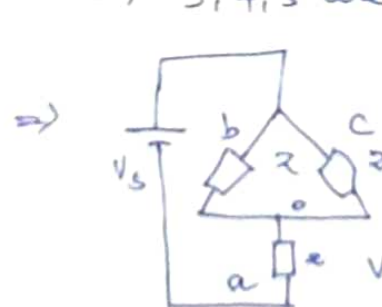
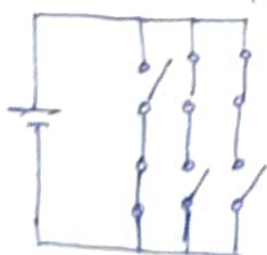
$$V_{oc} = \frac{V_s}{3}$$

$$V_{ab} = V_{ao} + V_{ob} = -\frac{V_s}{3} + \frac{2V_s}{3} = \frac{V_s}{3}$$

$$V_{bc} = V_{bo} + V_{oc} = \frac{2V_s}{3} + \frac{V_s}{3} = V_s$$

$$V_{ca} = V_{co} + V_{oa} = -\frac{V_s}{3} + \frac{V_s}{3} = 0$$

Mode (v); $240^\circ - 300^\circ$, 3, 4, 5 are conduct



$$V_{bo} = \frac{V_s}{3}$$

$$V_{co} = \frac{V_s}{3}$$

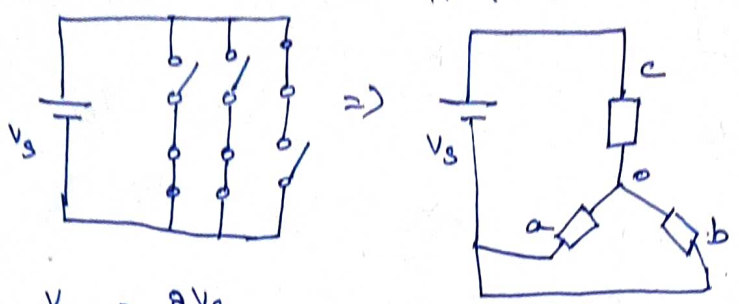
$$V_{ao} = \frac{2V_s}{3}$$

$$V_{ab} = V_{ao} + V_{ob} = -\frac{2V_s}{3} - \frac{V_s}{3} = -V_s$$

$$V_{bc} = V_{bo} + V_{oc} = \frac{V_s}{3} + (-\frac{V_s}{3}) = 0,$$

$$V_{ca} = V_{co} + V_{oa} = \frac{V_s}{3} + \frac{2V_s}{3} = V_s.$$

Mode (vi) : - 300° - 360°, 4, 5, 6 are conduct.



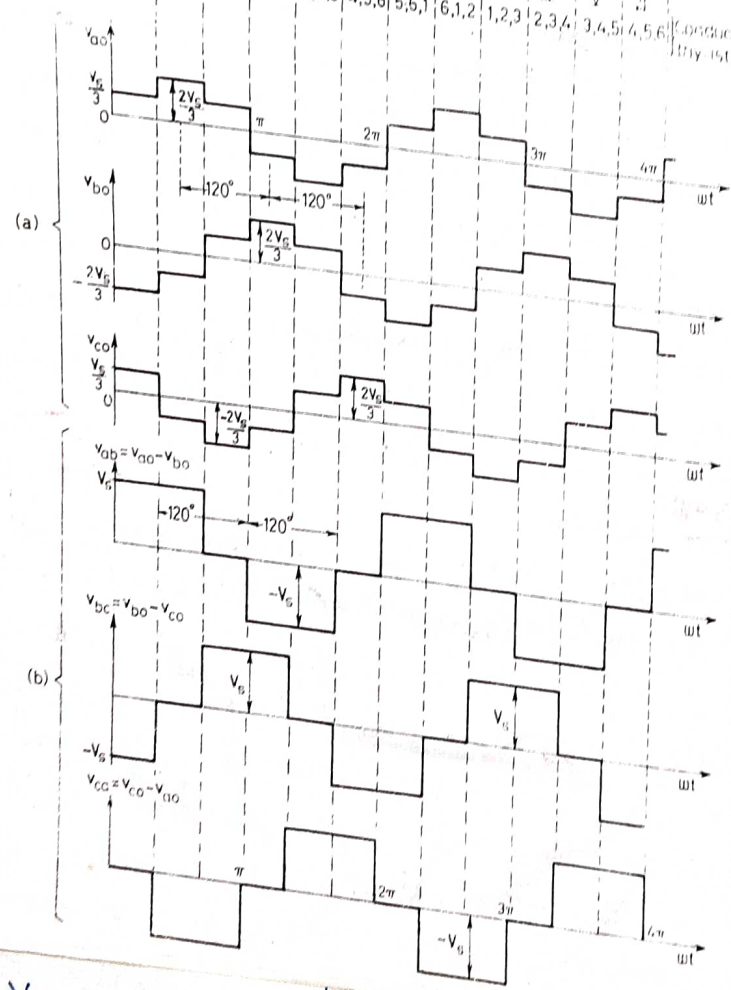
$$V_{co} = \frac{2V_s}{3}; \quad V_{oa} = \frac{V_s}{3}; \quad V_{ob} = \frac{V_s}{3}.$$

$$V_{ab} = V_{ao} + V_{ob} = -\frac{V_s}{3} + \frac{V_s}{3} = 0;$$

$$V_{bc} = V_{bo} + V_{oc} = -\frac{V_s}{3} - \frac{2V_s}{3} = -V_s.$$

$$V_{ca} = V_{co} + V_{oa} = \frac{2V_s}{3} + \frac{V_s}{3} = V_s.$$

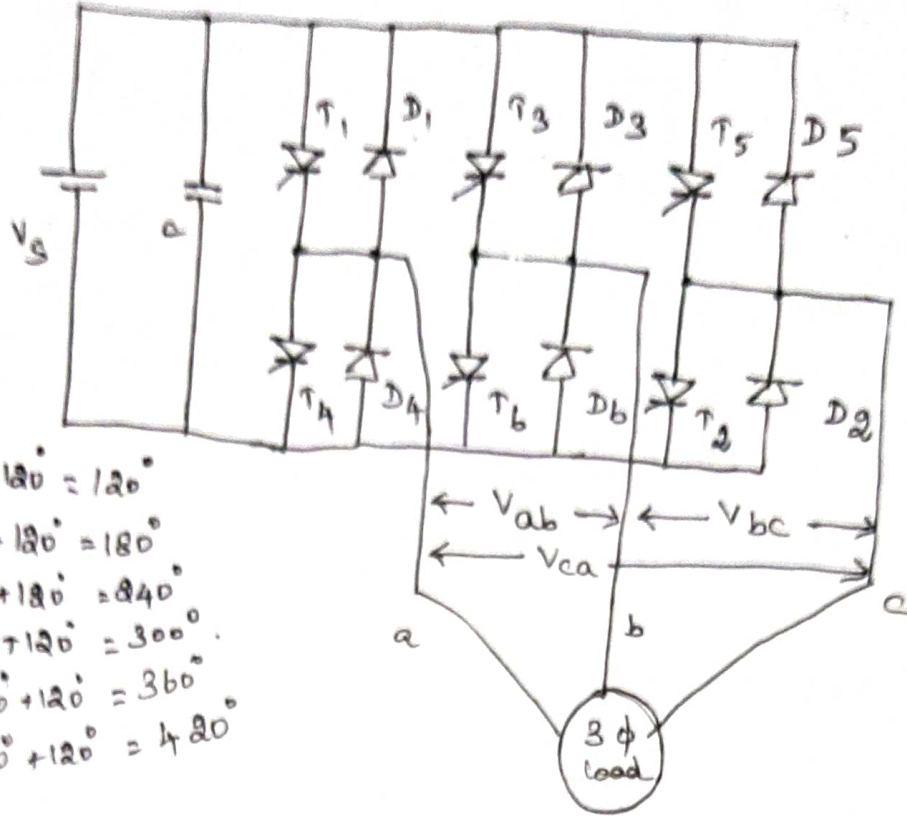
	-180°			180°																				
	T1		T4		T1		T4		T1		T4		T1		T4		T1		T4					
	T6		T3		T6		T3		T6		T3		T6		T3		T6		T3					
	T5		T2		T5		T2		T5		T2		T5		T2		T5		T2					
Steps	I	II	III	IV	V	VI	I	II	III	IV	V	VI	I	II	III	IV	V	VI	I	II	III	IV	V	VI
	5,6,1	6,1,2	1,2,3	2,3,4	3,4,5	4,5,6	5,6,1	6,1,2	1,2,3	2,3,4	3,4,5	4,5,6	5,6,1	6,1,2	1,2,3	2,3,4	3,4,5	4,5,6	5,6,1	6,1,2	1,2,3	2,3,4	3,4,5	4,5,6



$$V_{bc} = V_{bo} + V_{oc} = -\frac{2V_s}{3} - \frac{V_s}{3} = -V_s.$$

$$V_{ca} = V_{co} + V_{oa} = \frac{V_s}{3} - \frac{V_s}{3} = 0.$$

120° mode with star connected inverters (3φ).



$$T_1 = 0^\circ + 120^\circ = 120^\circ$$

$$T_2 = 60^\circ + 120^\circ = 180^\circ$$

$$T_3 = 120^\circ + 120^\circ = 240^\circ$$

$$T_4 = 180^\circ + 120^\circ = 300^\circ$$

$$T_5 = 240^\circ + 120^\circ = 360^\circ$$

$$T_6 = 300^\circ + 120^\circ = 420^\circ$$

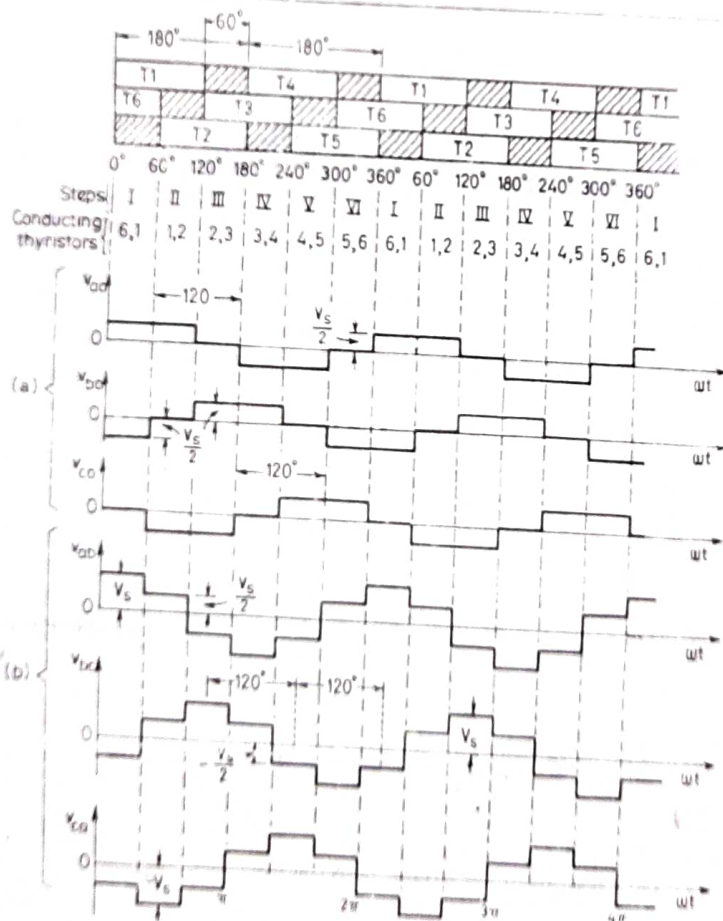
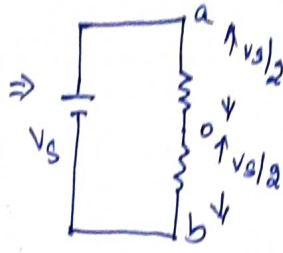
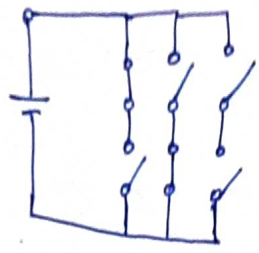


Fig 8.22 Voltage waveforms for 120° mode six-step 3-phase VSI

step I ; $0-60^\circ$, 1, 1 closed

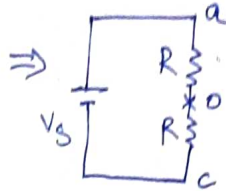
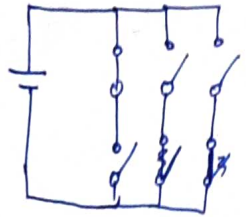


$$V_{ao} = V_s/2$$

$$V_{ob} = V_s/2$$

$$V_{oc} = 0$$

step II : $60-120^\circ$, 1, 2 closed



$$V_{ab} = V_{ao} + V_{ob} = V_s$$

$$V_{bc} = V_{bo} + V_{oc} = -V_s/2$$

$$V_{ca} = V_{co} + V_{oa} = -V_s/2$$

$$V_{ao} = V_s/2$$

$$V_{oc} = V_s/2$$

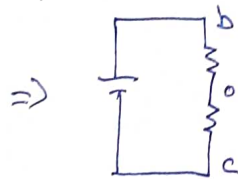
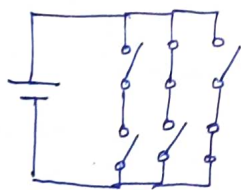
$$V_{bo} = 0$$

$$V_{ab} = V_{ao} + V_{ob} = V_s/2$$

$$V_{bc} = V_{bo} + V_{oc} = 0 + V_s/2 = V_s/2$$

$$V_{ca} = V_{co} + V_{oa} = -V_s/2 - V_s/2 = -V_s$$

step III :- $120-180^\circ$, 2, 3 closed



$$V_{ao} = 0$$

$$V_{bo} = V_s/2$$

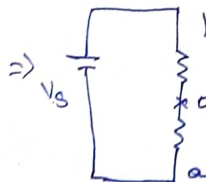
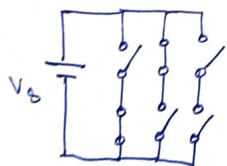
$$V_{oc} = V_s/2$$

$$V_{ab} = V_{ao} + V_{ob} = -V_s/2$$

$$V_{bc} = V_{bo} + V_{oc} = V_s/2 + V_s/2 = V_s$$

$$V_{ca} = V_{co} + V_{oa} = -V_s/2$$

step IV :- $180-240^\circ$, 3, 4 closed



$$V_{bo} = V_s/2$$

$$V_{oa} = V_s/2$$

$$V_{oc} = 0$$

$$V_{ab} = V_{ao} + V_{ob} = -V_s/2 - V_s/2 = -V_s$$

$$V_{bc} = V_{bo} + V_{oc} = V_s/2$$

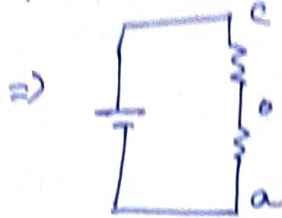
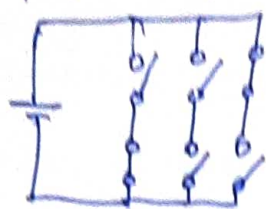
$$V_{ca} = V_{co} + V_{oa} = 0 + V_s/2 = V_s/2$$

val
ws
ms

$$V_{ca} = V_{co} + V_{oa} = -\frac{2V_s}{3} - \frac{V_s}{3} = -V_s$$

$$V_{ca} = V_{co} + V_{oa} = \frac{V_s}{3} - \frac{V_s}{3} = 0$$

Step \bar{v} : $240^\circ - 300^\circ$, 4, 5 closed.



$$V_{co} = V_s/2$$

$$V_{oa} = V_s/2$$

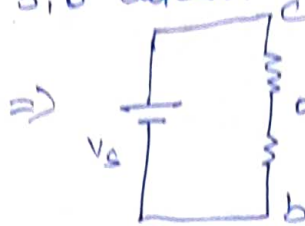
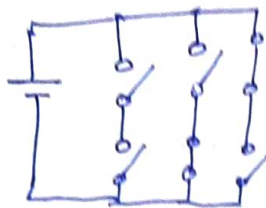
$$V_{ob} = 0.$$

$$V_{ab} = V_{ao} + V_{ob} = -V_s/2$$

$$V_{bc} = V_{bo} + V_{oc} = 0 - V_s/2 = -V_s/2$$

$$V_{ca} = V_{co} + V_{oa} = V_s/2 + V_s/2 = V_s.$$

Step \bar{v} : $300^\circ - 360^\circ$, 5, 6 closed.



$$V_{co} = V_s/2$$

$$V_{ob} = V_s/2.$$

$$V_{ao} = 0.$$

$$V_{ab} = V_{ao} + V_{ob} = V_s/2; \quad V_{bc} = V_{bo} + V_{oc} = -V_s/2 - V_s/2 = -V_s.$$

$$V_{ca} = V_{co} + V_{oa} = V_s/2 + 0 = V_s/2.$$

Fourier analysis of phase voltage waveform,

$$V_{ao} = \sum_{n=1,3,5}^{\infty} \frac{2V_s}{n\pi} \cos \frac{n\pi}{6} \sin n(\omega t + \pi/6).$$

$$V_{bo} = \sum_{n=1,3,5}^{\infty} \frac{2V_s}{n\pi} \cos \frac{n\pi}{6} \sin n(\omega t - \pi/6).$$

$$V_{co} = \sum_{n=1,3,5}^{\infty} \frac{2V_s}{n\pi} \cos \frac{n\pi}{6} \sin n(\omega t + 5\pi/6).$$

$$V_{ab} = \sum_{n=6k \pm 1}^{\infty} \frac{3V_s}{n\pi} \sin n(\omega t + \pi/3).$$

$$k = 0, 1, 2, 3, \dots$$

Step I ; $0-60^\circ$, $6, 1$ closed

Voltage control in 1 ϕ inverter :-

An ac load may require a constant input voltage. Any variations in the dc input voltage must be compensated in order to maintain a constant voltage at the a.c load terminals.

The various methods for the control of output voltage of inverters are as

- (i) External control of ac output voltage
- (ii) External control of dc input voltage
- (iii) Internal control of inverter.

External control of a.c output voltage :-

There are two possible methods: They are

- (i) AC Voltage control
- (ii) Series - inverter control.

(i) AC Voltage control :-

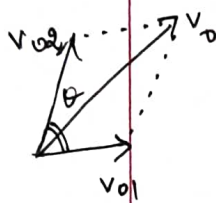


The voltage input to ac load is regulated through the firing angle control of ac voltage controller. This method gives rise to higher harmonic content in the output voltage.

$$V_s/3 - V_s/3 = 0$$

(b) series - Inverter control :-

In this method, the inverter output is fed to two transformers whose secondaries are connected in series. Phasor sum of the two fundamental voltages V_{o1} , V_{o2} gives the resultant fundamental voltage V_o . Here V_o is given by,



$$V_o = [V_{o1}^2 + V_{o2}^2 + 2 V_{o1} \cdot V_{o2} \cdot \cos \theta]^{1/2}$$

When θ is zero,

$$V_o = [V_{o1}^2 + V_{o2}^2 + 2 V_{o1} \cdot V_{o2}]^{1/2} \quad \therefore [\cos \theta = 1]$$

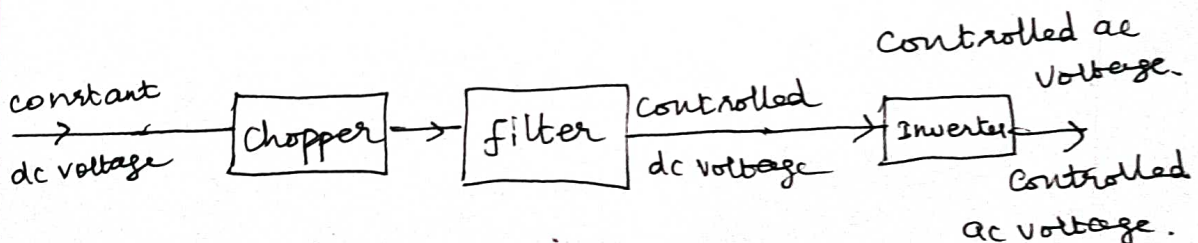
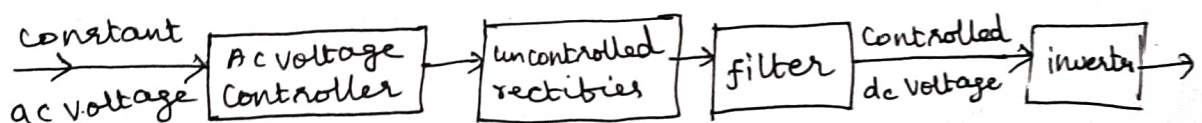
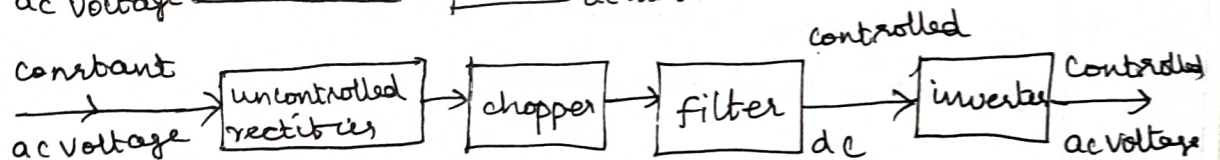
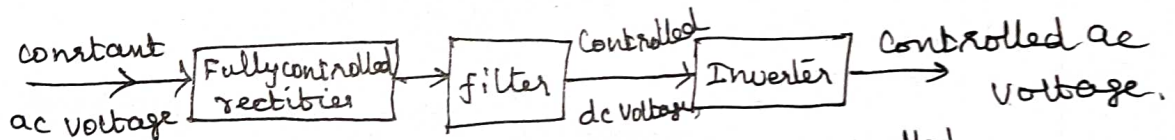
$$= [V_{o1} + V_{o2}]^{1/2}$$

$$= V_{o1} + V_{o2}$$

When $\theta = \pi$, $V_o = 0$. in case $V_{o1} = V_{o2}$.

The angle θ can be varied by the firing angle control of two inverters.

(2) External control of dc input voltage :-



step I ; $0-60^\circ$, 6, 1 closed

Voltage control in 1ϕ inverter :-

Disadvantages :-

- (i) Number of power converters are increased from two to three.
- (ii) For reducing ripple ~~current~~ ^{content}, filter circuit is required. This increase the cost, weight and size.
- (iii) Dc input decreased, commutating capacitor voltage also decreases.

(3) Internal control of Inverter :-

Pulse width modulation control :-

A fixed dc input voltage is given to the inverter and a controlled ac output voltage is obtained by adjusting the ON and OFF periods of the inverter components.

Advantages :-

- (i) Does not require any additional components.
- (ii) low order harmonics can be eliminated, filtering requirements are minimized.

Disadvantages :-

- (i) SCRs are expensive, they must possess low turn ON, turn OFF times.

~~(ii)~~

Harmonic Elimination and reduction in harmonics

by PWM :-

2) n^{th} harmonic can be eliminated by a proper choice of displacement angle β .

$$\sin n\beta/2 = 0$$

$$\beta = 360/n$$

3rd harmonic will be eliminated if,

$$\beta = \frac{360}{3} = 120^\circ$$

The Fourier series of output voltage can be expressed as,

$$V_o = \sum_{n=1,3,5}^{\infty} A_n \sin n\omega t$$

$$A_n = \frac{4V_s}{n\pi} \left[\int_0^{\alpha_1} \sin n\omega t \cdot d(\omega t) - \int_{\alpha_2}^{\alpha_1} \sin n\omega t \cdot d(\omega t) \right]$$

$$= \frac{4V_s}{\pi} \left[1 - 2\cos n\alpha_1 + 2\cos n\alpha_2 \right]$$

The 3rd & 5th harmonics would be eliminated, i.e. $A_3 = 0$.

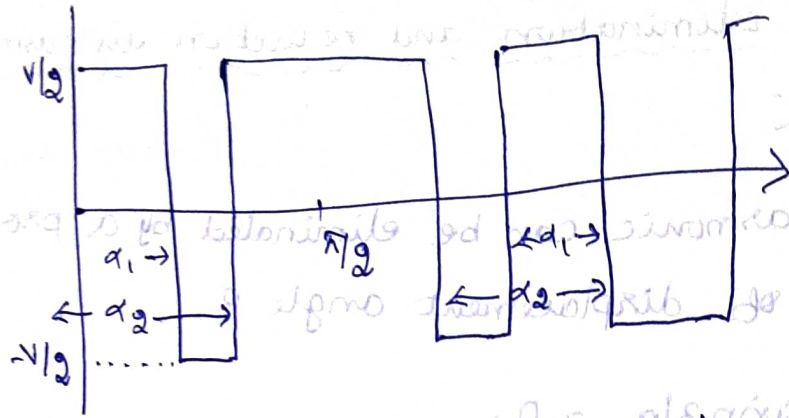
$$1 - 2\cos 3\alpha_1 + 2\cos 3\alpha_2 = 0 \quad \dots (1)$$

$$\alpha_1 = \frac{1}{3} \cos^{-1} (\cos 3\alpha_2 - 0.5) \quad \dots (1)$$

$$1 - 2\cos 5\alpha_1 + 2\cos 5\alpha_2 = 0$$

$$\alpha_1 = \frac{1}{5} \cos^{-1} (\cos 5\alpha_2 + 0.5) \quad \dots (2)$$

step I ; $0-60^\circ$, $6, 1$ closed

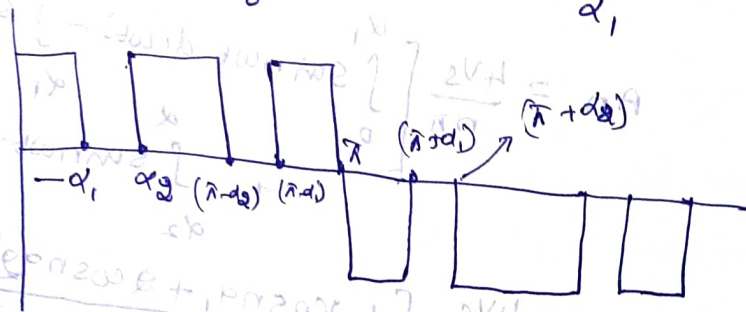


Equation (1) & (2) can be solved iteratively by assuming $\alpha_1 = 0$. repeating the calculations for α_1, α_2 . The result is $\alpha_1 = 23.62^\circ, \alpha_2 = 33.3^\circ$

$$A_n = \frac{4V_s}{n\pi} (1 - 2\cos n\alpha_1 + 2\cos n\alpha_2 - 2\cos n\alpha_3 + 2\cos n\alpha_4 - \dots) \quad (3)$$

with unipolar notches :-

$$A_n = \frac{4V_s}{\pi} \int_0^{\alpha_1} \sin n\omega t \cdot d\omega t + \int_{\alpha_1}^{\pi} \sin n\omega t \cdot d\omega t$$



$$= \frac{4V_s}{n\pi} (1 - \cos n\alpha_1 + \cos n\alpha_2)$$

5th, 3rd harmonics will be eliminated

$$1 - \cos 3\alpha_1 + \cos 3\alpha_2 = 0$$

$$1 - \cos 5\alpha_1 + \cos 5\alpha_2 = 0$$

$$\alpha_1 = 17.83, \alpha_2 = 37.97^\circ$$

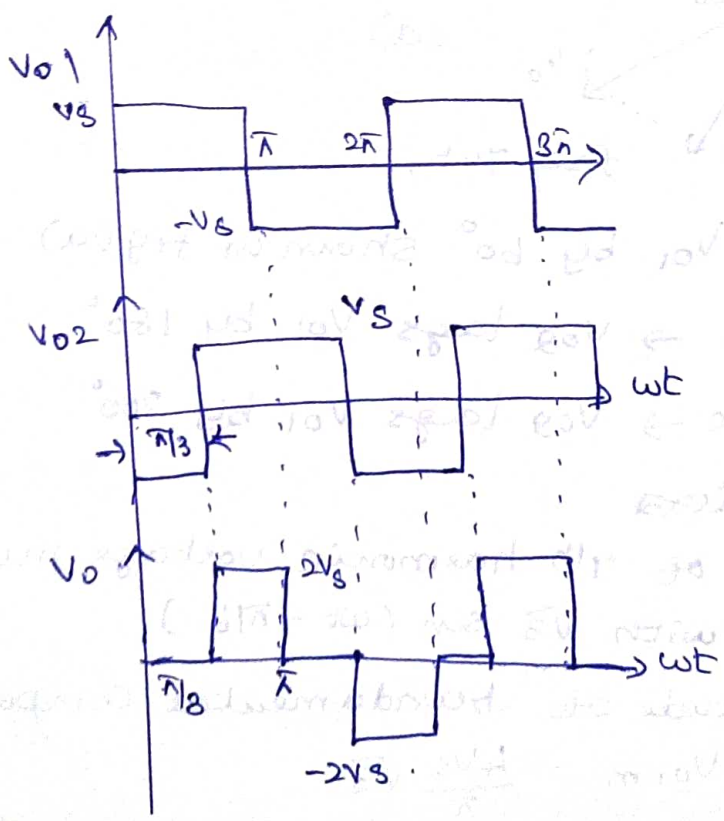
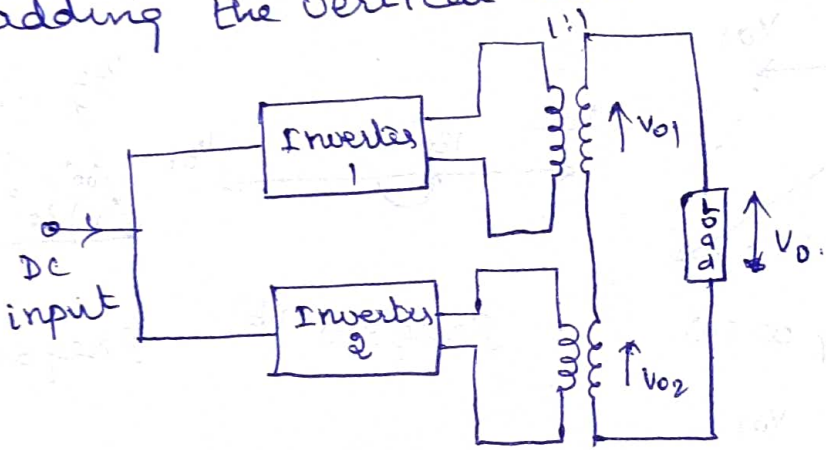
Harmonic Reduction by Transformer Connections :-

Output voltage from two or more inverters can be combined by means of transformers to get a net output voltage with reduced harmonic content. Two transformers are in series.

V_{o1} \rightarrow from inverter 1

V_{o2} \rightarrow from inverter 2

V_{o2} have a phase shift of $\pi/3$ radians with respect to V_{o1} . The resultant voltage V_o obtained by adding the vertical coordinates of V_{o1} & V_{o2} .



step I ; 0-60°, b, 1 closed

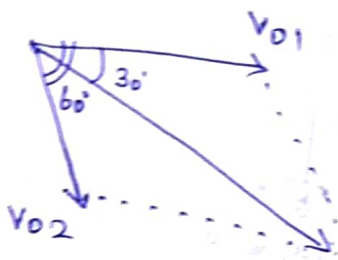
$$V_{o1} = \frac{4V_s}{\pi} \left[\sin \omega t + \frac{1}{3} \sin 3\omega t + \frac{1}{5} \sin 5\omega t + \frac{1}{7} \sin 7\omega t + \dots \right]$$

$$V_{o2} = \frac{4V_s}{\pi} \left[\sin(\omega t - \pi/3) + \frac{1}{3} \sin 3(\omega t - \pi/3) + \frac{1}{5} \sin 5(\omega t - \pi/3) + \frac{1}{7} \sin 7(\omega t - \pi/3) + \dots \right]$$

resultant voltage V_o is,

$$V_o = V_{o1} + V_{o2}$$

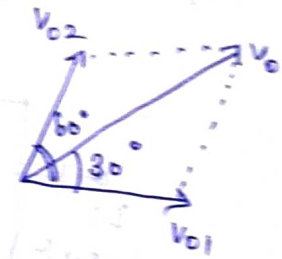
$$= \frac{4V_s}{\pi} \sqrt{3} \left[\sin(\omega t - \pi/6) + \frac{1}{5} \sin(5\omega t + \pi/6) + \frac{1}{7} \sin(7\omega t - \pi/6) + \dots \right]$$



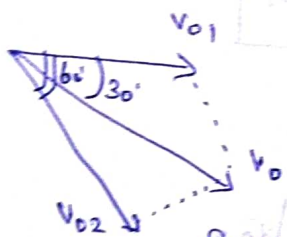
freq ω (a)



freq 3ω (b)



freq 5ω (c)



freq 7ω (d)

fundamental frequency $\Rightarrow V_{o2}$ lags V_{o1} by 60° shown in fig (a).

3rd harmonic $\rightarrow V_{o2}$ lags V_{o1} by 180° .

5th harmonic $\rightarrow V_{o2}$ lags V_{o1} by 300° .

7th harmonic \rightarrow

resultant of n th harmonic voltage must be associated with $\sqrt{3} \sin(\omega t - \pi/6)$.

The amplitude of fundamental component of V_o ,

$$V_{o1m} = \frac{4V_s}{\pi} \sqrt{3}$$

Space Vector Modulation :-

Space vector approach to PWM involves the use of voltage space vectors as reference, instead of 3 ϕ modulating waves. It considers combined effect of all 3 ϕ voltages.

At steady state the voltage space vector has a constant magnitude and revolves with constant frequency. The direction of rotation depends on the phase sequence.

SVPWM is used for inverter fed drives because of its superior harmonic quality and extended linear range of operation.

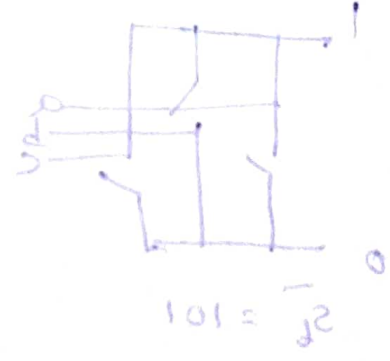
If a 3 ϕ windings displaced in space by 120 $^\circ$ are excited by 3 ϕ currents with a phase difference of 120 $^\circ$ a magnetic field rotating in space will be generated.

Line to line voltages

$$V_{ab} = V_{aN} - V_{bN}$$

$$V_{bc} = V_{bN} - V_{cN}$$

$$V_{ca} = V_{cN} - V_{aN}$$



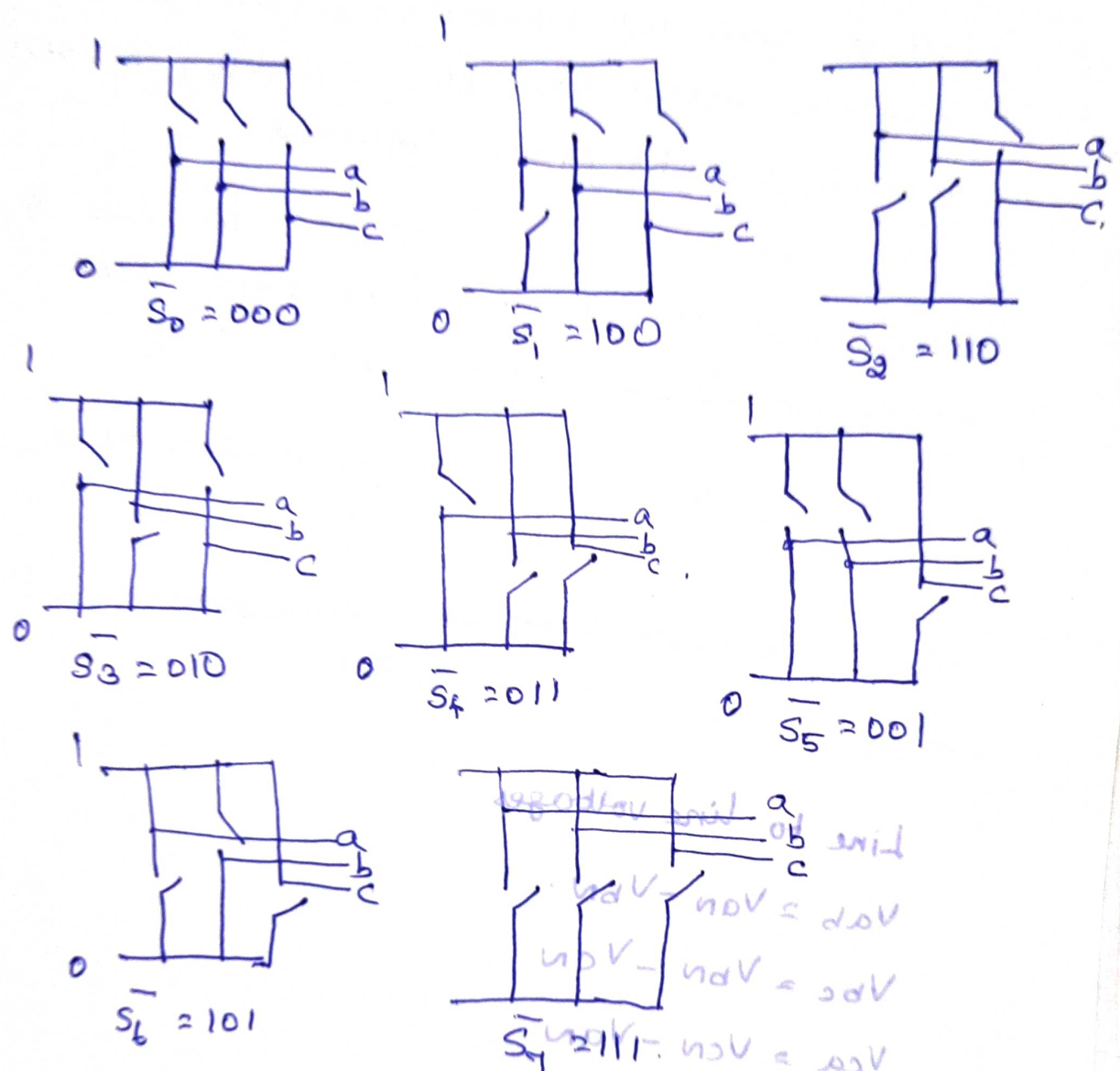
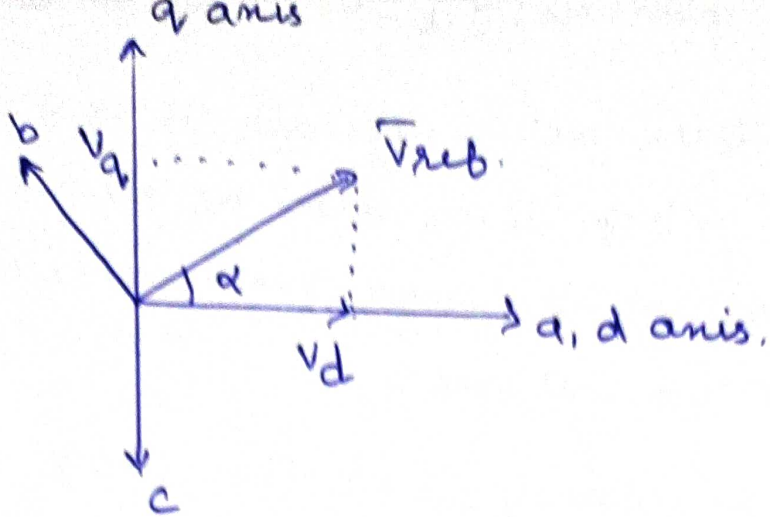
Phase voltages

$$V_{aN} = \frac{2}{3} V_{aN} - \frac{1}{3} V_{bN} - \frac{1}{3} V_{cN}$$

$$V_{bN} = -\frac{1}{3} V_{aN} + \frac{2}{3} V_{bN} - \frac{1}{3} V_{cN}$$

$$V_{cN} = -\frac{1}{3} V_{aN} - \frac{1}{3} V_{bN} + \frac{2}{3} V_{cN}$$

000	000
001	001
010	010
011	011
100	100
101	101
110	110
111	111
000	000



Possible switching

State	Vector
S_1	100
S_2	110
S_3	010
S_4	011
S_5	001
S_6	101
S_7	111
S_0	000

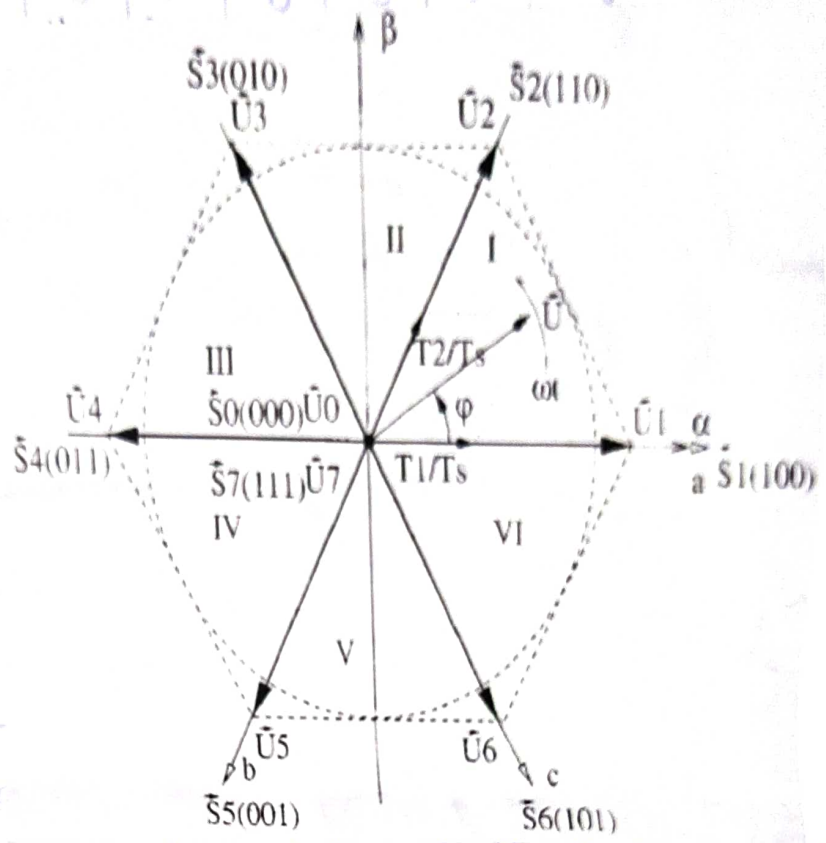
$$V_{a0} = \frac{2}{3} V_{dc}$$

$$V_{b0} = -\frac{1}{3} V_{dc} + \frac{1}{3} V_{dc} = 0$$

$$V_{c0} = -\frac{1}{3} V_{dc} - \frac{1}{3} V_{dc} = -\frac{2}{3} V_{dc}$$

In the vector spacing, according to the equivalence principle, the following operation rules are obeyed:

$U_1 = -U_4$
 $U_2 = -U_5$
 $U_3 = -U_6$, $U_7 = U_0 = 0$, $U_1 + U_3 + U_5 = 0$



In one sampling interval, the output voltage vector can be written as:

$$\vec{U}(k) = \frac{E_0}{T_s} \vec{U}_0 + \frac{E_1}{T_s} \vec{U}_1 + \dots + \frac{E_7}{T_s} \vec{U}_7$$

The decomposition of U into $U_1, U_2, U_3, U_4, U_5, U_6, U_7$ has infinite ways. In order to reduce the number of switching actions and make full use of active turn on time for space vectors,

vector U can be expressed as,

$$\vec{U} = \frac{T_1}{T_s} \vec{U}_1 + \frac{T_2}{T_s} \vec{U}_2 + \frac{T_7}{T_s} \vec{U}_7 + \frac{T_0}{T_s} \vec{U}_0$$

where $T_s = T_1 + T_2 = T_0 + T_7 \geq 0$,

$T_7 \geq 0$ & $T_0 \geq 0$.

voltage vectors	switching vectors			Line to neutral voltage			Line to line voltage		
	a	b	c	V_{an}	V_{bn}	V_{cn}	V_{ab}	V_{bc}	V_{ca}
v_0	0	0	0	0	0	0	0	0	0
v_1	1	0	0	$2/3$	$-1/3$	$-1/3$	1	0	-1
v_2	1	1	0	$1/3$	$1/3$	$-2/3$	0	1	-1
v_3	0	1	0	$-1/3$	$2/3$	$-1/3$	-1	1	0
v_4	0	1	1	$-2/3$	$1/3$	$1/3$	-1	0	1
v_5	0	0	1	$-1/3$	$-1/3$	$2/3$	0	-1	1
v_6	1	0	1	$1/3$	$-2/3$	$1/3$	1	-1	0
v_7	1	1	1	0	0	0	0	0	0

Applications 1.4 - Realization of Space Vector PWM

Step 1: Determine V_d , V_q , V_{ref} and angle.

Step 2: Determine time duration T_1 , T_2 , T_0 .

Step 3: Determine the switching time of each transistor (S_1 to S_6).

Co-ordinate transformation: abc to dq, for inverter work. In order to reduce the number of switching actions and more full use of active turn on time for space vectors, vector U can be performed as:

$$\sum_{0}^6 \frac{0T}{2T} + \sum_{1}^2 \frac{1T}{2T} + \sum_{2}^4 \frac{1T}{2T} + \sum_{4}^6 \frac{1T}{2T} = \sum_{0}^6 \frac{1T}{2T}$$

Voltage control of single phase inverter :-

- i) single pulse width modulation,
 - ii) Multiple pulse width modulation,
 - iii) sinusoidal pulse width modulation,
 - iv) Modified sinusoidal PWM,
 - v) phase displacement control.
- The methods are applicable to 3 ϕ inverter.

i) Single pulse width modulation :-

Only one pulse per half cycle and the output rms voltage is changed by varying the width of the pulse. The gating signals are generated by comparing the rectangular control signal of amplitude A_r with triangular carrier signal A_c .

$$\text{Modulation index } M = \frac{A_r}{A_c}$$

$$\text{RMS value of output voltage } V_{or} = \left[\frac{1}{\pi} \int_{\frac{\pi-\delta}{2}}^{\frac{\pi+\delta}{2}} V_s^2 dt \right]^{\frac{1}{2}}$$

Fourier series of output voltage,

$$V_o = \sum_{1,3,5} (A_n \cos n\omega t + B_n \sin n\omega t)$$

Half wave symmetry, $a_0 = a_n = 0$.

$$\begin{aligned} B_n &= \frac{2}{\pi} \int_0^{\pi} V_s \sin n\omega t \cdot dt = \frac{2}{\pi} \int_{\frac{\pi-\delta}{2}}^{\frac{\pi+\delta}{2}} V_s \sin n\omega t \cdot dt \\ &= \frac{2V_s}{\pi} \left(\frac{-\cos n\omega t}{n} \right)_{\frac{\pi-\delta}{2}}^{\frac{\pi+\delta}{2}} = \frac{2V_s}{n\pi} (\cos n\omega t)_{\frac{\pi-\delta}{2}}^{\frac{\pi+\delta}{2}} \\ &= \frac{2V_s}{n\pi} \left[\cos n \left(\frac{\pi-\delta}{2} \right) - \cos n \left(\frac{\pi+\delta}{2} \right) \right] \end{aligned}$$

step I ; $0-60^\circ$, b, l closed

$$V_o = \sum_{1,3,5} \frac{AV_s}{n\pi} \sin \frac{n\delta}{2} \sin n\omega t$$

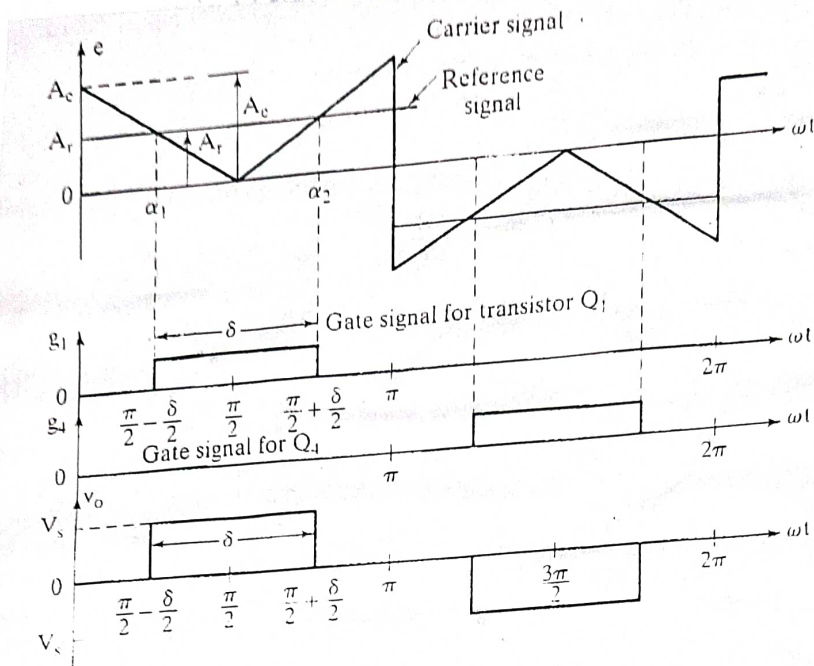


Fig 1.7 Single-Pulse-Width-Modulation

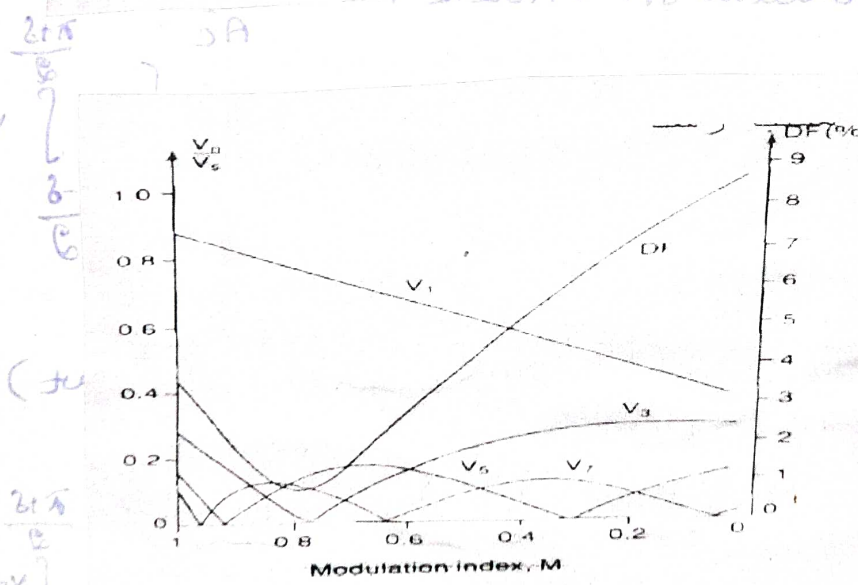


Fig 1.8 Harmonic profile

$$\left[\left(\frac{2+\pi}{\delta} \right) n 2\omega - \left(\frac{2-\pi}{\delta} \right) n 2\omega \right] \frac{2V_s}{n\pi}$$

Multiple pulse width Modulation :-

In multiple PWM control, instead of having a single pulse per half cycle, there will be multiple number of pulses per half cycle all of them being of equal width.

$f_o = f_r$ output frequency is determined by frequency of reference signal.

f_c determines no. of pulses / half cycle.

$$\text{No. of pulses / half cycle} = p = \frac{f_c}{2f_o} = \frac{m_f}{2}$$

$m_f \rightarrow$ frequency modulation ratio.

$M \rightarrow$ varied from 0 to 1

pulse width 0 to π/p .

voltage 0 to V_s .

$$\text{Output RMS Voltage } V_{or} = \left[\frac{1}{\pi/p} \int_0^{\pi/p + \delta} V_s^2 \cdot d\omega t \right]^{1/2}$$

$$= V_s \sqrt{\frac{p\delta}{\pi}}$$

Instantaneous output voltage,

Half wave symmetry $\Rightarrow a_0 = a_n = 0$.

$$b_n = \frac{V_s}{\pi} \left[\int_{d_m}^{d_m + \delta} \cos n\omega t \cdot d\omega t - \int_{\pi + d_m}^{\pi + d_m + \delta} \cos n\omega t \cdot d\omega t \right]$$

$$= \frac{V_s}{\pi} \left[\left(\frac{\sin n\omega t}{n} \right)_{d_m}^{d_m + \delta} - \left(\frac{\sin n\omega t}{n} \right)_{\pi + d_m}^{\pi + d_m + \delta} \right]$$

$$= \frac{V_s}{n\bar{n}} \left[\sin(n(\omega t + \delta)) - \sin(n\omega t) - \sin(n(\bar{n} + \omega t + \delta)) + \sin(n(\bar{n} + \omega t)) \right]$$



For a two pulse,

$$A_n = \frac{2}{n} \int_0^{\omega} V_s \sin n\omega t \cdot d\omega t$$

$$= \frac{2}{n} \int_{\omega - d/2}^{\omega + d/2} V_s \sin n\omega t \cdot d\omega t \times 2$$

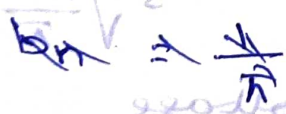
$$= \frac{4V_s}{n\bar{n}} (\cos n\omega t)$$

$$= \frac{4V_s}{n\bar{n}} \left[\cos n(\omega - d/2) - \cos n(\omega + d/2) \right]$$

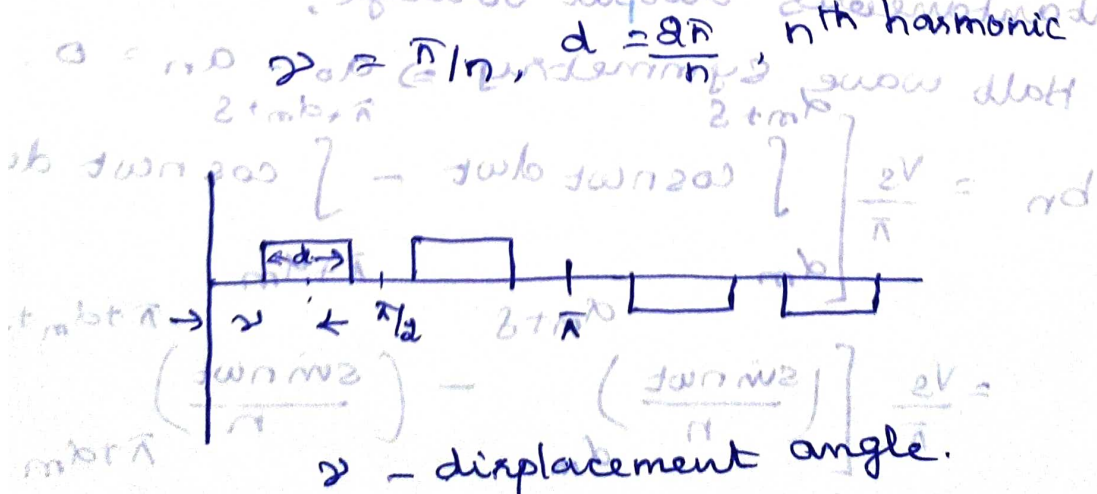
$$= \frac{4V_s}{n\bar{n}} \left[\cos n\omega \cos nd/2 + \sin n\omega \sin nd/2 - \cos n\omega \cos nd/2 + \sin n\omega \sin nd/2 \right]$$

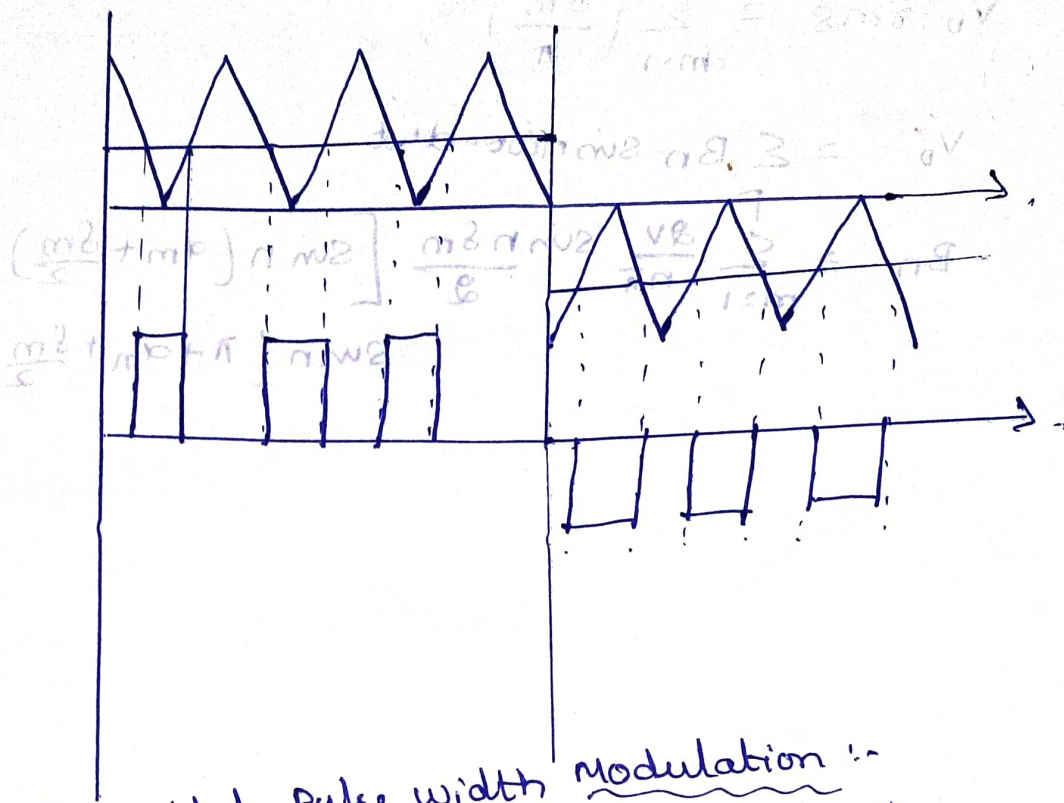
$$= \sum \frac{8V_s}{n\bar{n}} \sin n\omega \sin nd/2 \cdot \sin n\omega t \quad (n=1, 3, 5, \dots)$$

$$\Rightarrow \frac{8V_s}{n\bar{n}} \sin n\omega \sin nd/2 \cdot \sin n\omega t \quad (n=1, 3, 5, \dots)$$



n th harmonic eliminated.





Sinusoidal Pulse width Modulation :-

⇒ width of pulses are varied in proportion to

the amplitude of sine wave.

$$A_n = \frac{2}{\pi} \int_{\alpha_m - \delta/2}^{\alpha_m + \delta/2} v_s \sin \omega t \cdot d\omega t$$

$$= \sum_{n=1}^{\infty} \frac{4 E_{dc}}{n\pi} \sin \frac{n\delta}{2} \sin n\alpha_m$$

$$B_n = \frac{2}{\pi} \int_{\alpha_m - \delta/2}^{\alpha_m + \delta/2} \cos n\omega t \cdot d\omega t$$

$$B_n = \sum_{n=1}^{\infty} \frac{4 E_{dc}}{n\pi} \cos \frac{n\delta}{2} \sin n\alpha_m$$

If the amplitude of sine wave ↓, output voltage ↓

$$M = \frac{A_r}{A_c}$$

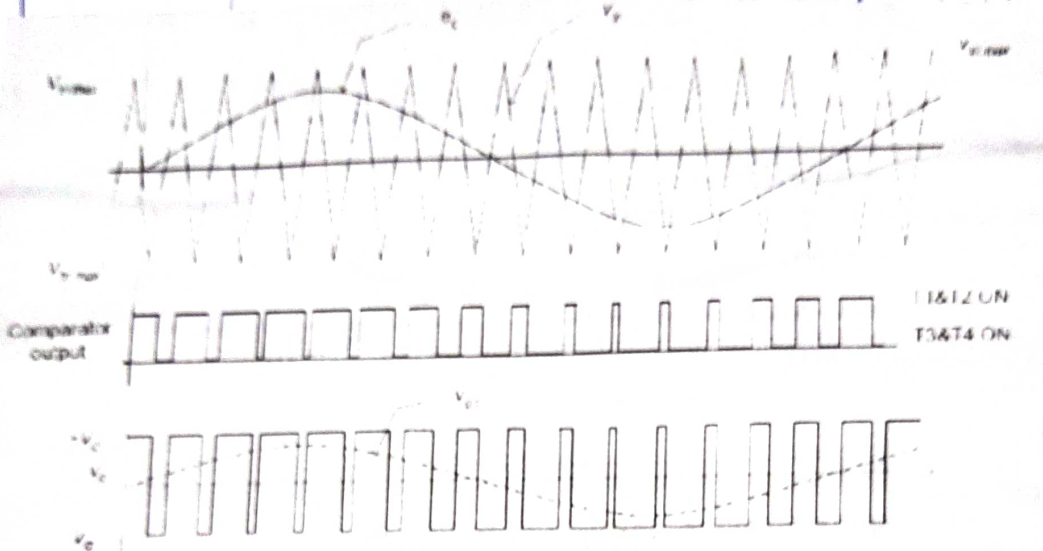
$$\text{Frequency Modulation} = \frac{\omega_c}{\omega_r} = \frac{f_c}{f_r}$$

step I ; 0-60°, 6, 1 closed

$$V_{0, rms} = \sum_{m=1}^p \left(\frac{\delta_m}{\pi} \right)^{1/2}$$

$$V_0 = \sum B_n \sin n\omega t \cdot d \cdot \omega t$$

$$B_n = \sum_{m=1}^p \frac{dv}{n\pi} \sin n \delta_m \left[\sin n \left(\alpha_m + \frac{\delta_m}{2} \right) - \sin n \left(\pi + \alpha_m + \frac{\delta_m}{2} \right) \right]$$



Modified sinusoidal pulse width modulation

⇒ Near the peak of sine wave, the pulse width does not change, with variation in modulation index.

⇒ The carrier wave is applied only during (0-60° & 180° to 180°).

Advantages :-

- i) Fundamental component increased.
- ii) Harmonics reduced.
- iii) Switching losses are reduced.
- iv) Reduced number of switching devices.

$\frac{V_0}{V_c} = M$
 Modulation index

① compare CSI and VSI

CSI

① Most commonly used for synchronous motor control

② Peak current rating is smaller

③ Response time is less

VSI

Most commonly used for Induction Motor control

Peak current rating higher.

good response time

② what is duty cycle?

duty cycle can be produced by the comparison of d.c reference signal with the carrier signal.

V_{ref} → amplitude of dc reference signal

V_c → amplitude of carrier signal.

$$S = \frac{V_{ref}}{V_c}$$

The ratio of the reference signal to carrier signal gives the modulation index.

③ why thyristors are not preferred for Inverter?
Thyristors require extra commutation circuit for turn off which result in increased complexity of the circuit. For this reason thyristors are not preferred for inverters.

④ what is a matrix converter?
Converters built on the bi-directional, bipolar switches are called matrix converters. They provide a direct power flowing between n-phase ac source and m-phase load.

⑤ what are the disadvantages of the harmonics present in the inverter system?

i) The output voltage and waveform becomes distorted one.

ii) Switching losses is increased.

⑥ what are the possible methods to control a.c output voltage.

i) AC voltage control.

ii) Series - inverter control.

UNIT-IV INVERTERS

1. Why diodes should be connected in antiparallel with the thyristors in inverter circuits?

For RL loads, load current will not be in phase with load voltage and the diodes connected in anti parallel will allow the current to flow when the main thyristors are turned off. These diodes are called feedback diodes.

2. What types of inverters require feedback diodes?

VSI with RL load

3. What is meant a series inverter?

An inverter in which the commutating elements are connected in series with the load is called a series inverter.

4. What is the condition to be satisfied in the selection of L and C in a series inverter?

$$R^2 < 4L$$

5. What is meant a parallel inverter?

An inverter in which the commutating elements are connected in parallel with the load is called a parallel inverter.

6. What are the applications of a series inverter?

The thyristorised series inverter produces an approximately sinusoidal waveform at a high output frequency, ranging from 200 Hz to 100kHz. It is commonly used for fixed output applications such as a. Ultrasonic generator. b. Induction heating. c. Sonar Transmitter d. Fluorescent lighting.

7. How is the inverter circuit classified based on commutation circuitry?

a. Line commutated inverters. b. Load commutated inverters. c. Self commutated inverters. d. Forced commutated inverters.

8. What is meant by McMurray inverter?

It is an impulse commutated inverter which relies on LC circuit and an auxiliary thyristor for commutation in the load circuit.

9. What are the applications of a CSI?

a. Induction heating b. Lagging VAR compensation c. Speed control of ac motors d. Synchronous motor starting.

10. What is meant by PWM control?

In this method, a fixed dc input voltage is given to the inverter and a controlled ac

output voltage is obtained by adjusting the on and off periods of the inverter components. This is the most popular method of controlling the output voltage and this method is termed as PWM control .

11. What are the advantages of PWM control?

- a. The output voltage can be obtained without any additional components.
- b. Lower order harmonics can be eliminated or minimized along with its output voltage control. As the higher order harmonics can be filtered easily, the filtering requirements are minimized.

12. What are the disadvantages of the harmonics present in the inverter system?

- a. Harmonic currents will lead to excessive heating in the induction motors. This will reduce the load carrying capacity of the motor.
- b. If the control and the regulating circuits are not properly shielded, harmonics from power ride can affect their operation and malfunctioning can result.
- c. Harmonic currents cause losses in the ac system and can even some time produce resonance in the system. Under resonant conditions, the instrumentation and metering can be affected.
- d. On critical loads, torque pulsation produced by the harmonic current can be useful.

13. What are the methods of reduction of harmonic content?

- a. Transformer connections
- b. Sinusoidal PWM
- c. Multiple commutation in each cycle
- d. Stepped wave inverters

15. What are the disadvantages of PWM control?

SCRs are expensive as they must possess low turn-on and turn-off times.

16. What does ac voltage controller mean?

It is device which converts fixed alternating voltage into a variable voltage without change in frequency.

17. What are the applications of ac voltage controllers?

- a. Domestic and industrial heating
- b. Lighting control
- c. Speed control of single phase and three phase ac motors
- d. Transformer tap changing

18. What are the advantages of ac voltage controllers?

- a. High efficiency
- b. Flexibility in control
- c. Less maintenance

19. What are the disadvantages of ac voltage controllers?

The main draw back is the introduction of harmonics in the supply current and the load voltage waveforms particularly at low output voltages .

20. What are the two methods of control in ac voltage controllers?

- a. ON-OFF control
- b. Phase control

9) Why the THD has to be mitigated?

- ① To improve power factor and reduce system loss.
- ② Minimise interference with other equipment.
- ③ To improve system voltage/current waveform.
- ④ To prevent nuisance tripping of fuse and circuit breakers.

10) What are the purposes of free back diodes in Inverters.
For inductive load, current i_o will not be in phase with voltage v_o and diodes connected in antiparallel with thyristors will allow the current to flow when the main thyristors are turned off. These diodes are called free back diodes.

11) Mention the PWM methods in Inverters.

- i) Single pulse modulation.
- ii) Multiple pulse modulation.
- iii) Sinusoidal pulse modulation.
- iv) Modified sinusoidal pulse width modulation.

12) What are the advantages and disadvantages of resonant pulse converter?

Advantages :-

- i) Switching losses are less.
- ii) Less electromagnetic interference.
- iii) Operating switching frequency is high.
- iv) Efficiency is high.

Disadvantages :

- 1) Limited frequency.
- 2) Larger size.
- 3) Heavy weight.
- 4) Power dissipation may occur in any working condition.

UNIT - V

29

single phase and 3 ϕ AC voltage controllers - control strategy - Power factor control - Multistage sequence control - single phase and three phase cycloconverters - Introduction to Matrix converters.

1 ϕ AC voltage controllers :

AC voltage controllers are semiconductor based circuits which convert fixed alternating voltage to variable alternating voltage directly without change in the frequency.

Applications :

- i) Domestic and Industrial heating
- ii) Transformer tap changing
- iii) Lighting control
- iv) Speed control drives
- v) starting of Induction motors :

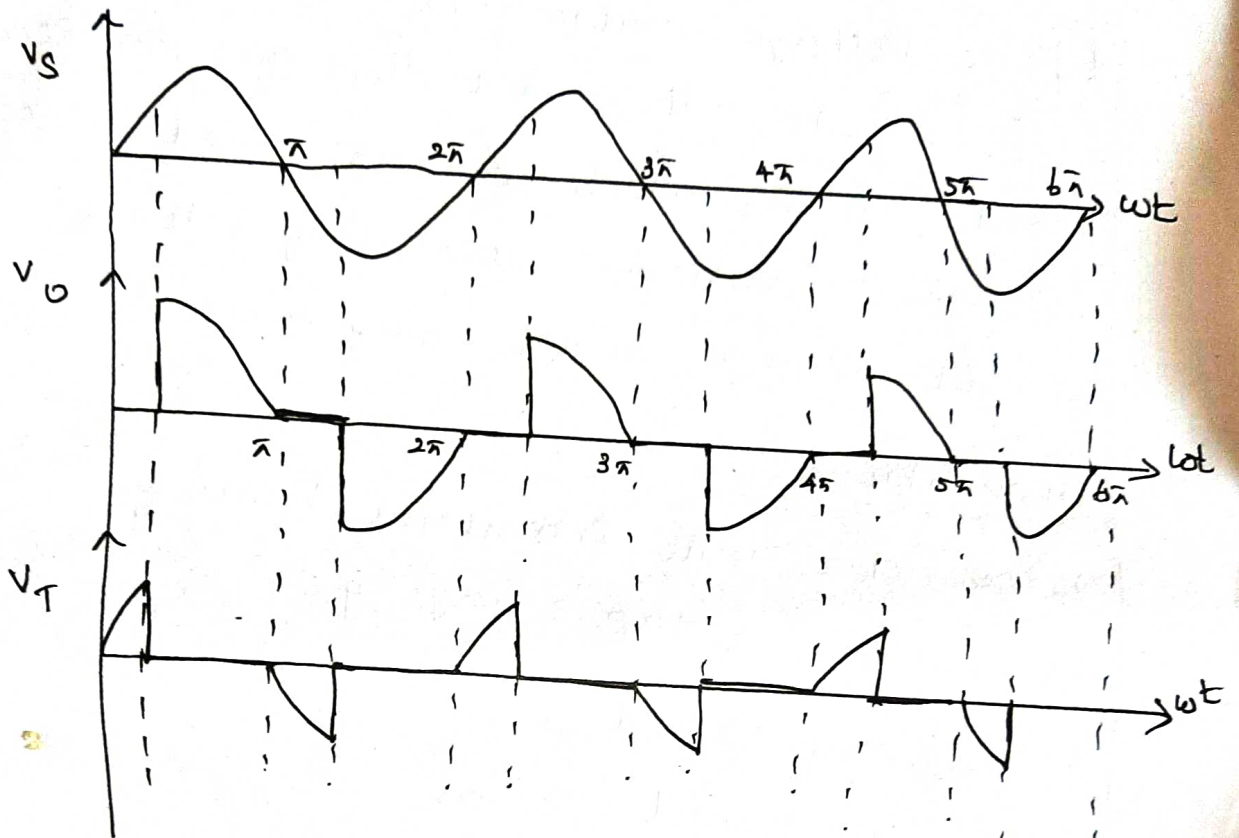
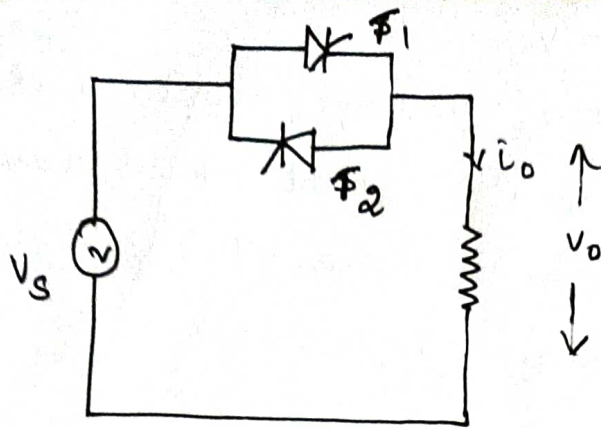
single phase AC voltage controller with resistive load :

Phase control :-

It consists of two SCRs connected in antiparallel. So, it is possible to have current flow in either direction.

During +ve half cycle, T_1 is triggered at $\omega t = \alpha$, it conducts from $\omega t = \alpha$ to π ,

During -ve half cycle, T_2 is triggered at $\omega t = \pi + \alpha$, it conducts from $\omega t = \pi + \alpha$ to 2π .



RMS Voltage :

$$V_r = \sqrt{\frac{1}{\pi} \int_{\alpha}^{\pi} v_m^2 \sin^2 \omega t \cdot d\omega t}$$

$$= \left[\frac{v_m^2}{2\pi} \int_{\alpha}^{\pi} (1 - \cos 2\omega t) \cdot d\omega t \right]^{1/2}$$

$$= \left[\frac{v_m^2}{2\pi} \left[\omega t - \frac{\sin 2\omega t}{2} \right]_{\alpha}^{\pi} \right]^{1/2}$$

$$= \frac{v_m^2}{2\pi} \left[\pi - \frac{\sin 2\pi}{2} - \alpha + \frac{\sin 2\alpha}{2} \right]^{1/2}$$

$$V_r = \frac{v_m}{\sqrt{2\pi}} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right)^{1/2}$$

RMS load current

$$I_r = \frac{V_r}{R}$$

$$= \frac{V_m}{\sqrt{2\pi} R} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right)^{1/2}$$

Power factor of the load :-

$$= \frac{V_r^2 / R}{V_s \cdot I_s}$$

$$= \frac{V_r^2 / R}{V_s \times V_r / R} = \frac{V_r}{V_s}$$

$$= \frac{\frac{V_m}{\sqrt{2\pi}} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right)^{1/2}}{V_m / \sqrt{2}}$$

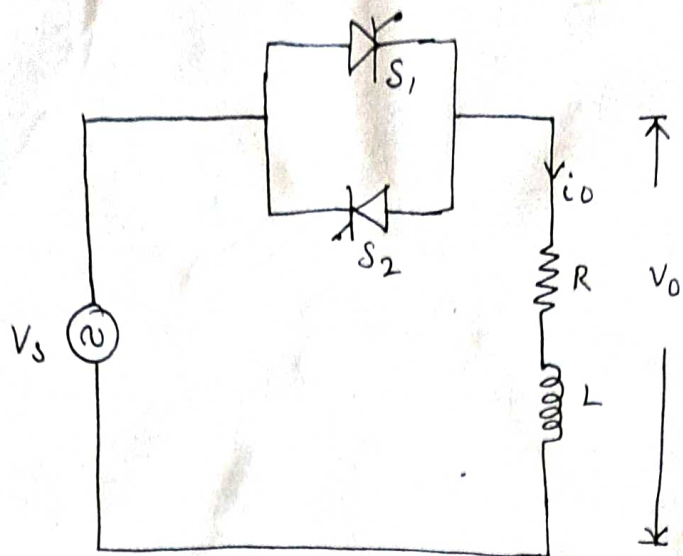
$$P.F = \left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi} \right]^{1/2}$$

1 ϕ AC voltage controller with RL load :-

- \Rightarrow During +ve half cycle, $\omega t = 0$ to π , T_1 is forward biased, $\omega t = \alpha$, T_1 is triggered.
- \Rightarrow At $\omega t = \pi$, current is not zero, T_1 is reverse biased, but does not turn off because of i_o .
- \Rightarrow From β to $\pi + \alpha$, no current exist in the power circuit.
- \Rightarrow T_2 turned on at $(\pi + \alpha) > \beta$, $i_o = I_{T_2}$.

β is called extinction angle

[operation in page no : 6] \rightarrow P.T.O.



expression for load current (i_o) :-

KVL for the circuit, when T_1 conducts,

$$V_s = V_m \sin \omega t = Ri_o + L \cdot \frac{di_o}{dt} \dots \alpha < \omega t < \beta$$

solution of this equation is,

$$i_o = \frac{V_m}{Z} \sin(\omega t - \phi) + A \cdot e^{-(R/L)t} \dots \text{--- (I)}$$

$$Z = [R^2 + \omega L^2]^{1/2}; \quad \phi = \tan^{-1} \omega L/R$$

Boundary conditions are, $\omega t = \alpha$, $t = \alpha/\omega$; $i_o = 0$.

$$0 = \frac{V_m}{Z} \sin(\alpha - \phi) + A e^{-R\alpha/L\omega}$$

$$A = -\frac{V_m}{Z} \sin(\alpha - \phi) e^{R\alpha/L\omega} \dots \text{--- (II)}$$

sub II in (I)

$$i_o = \frac{V_m}{Z} \sin(\omega t - \phi) - \frac{V_m}{Z} \sin(\alpha - \phi) \cdot e^{R/L(\alpha/\omega - t)}$$

RMS output voltage :

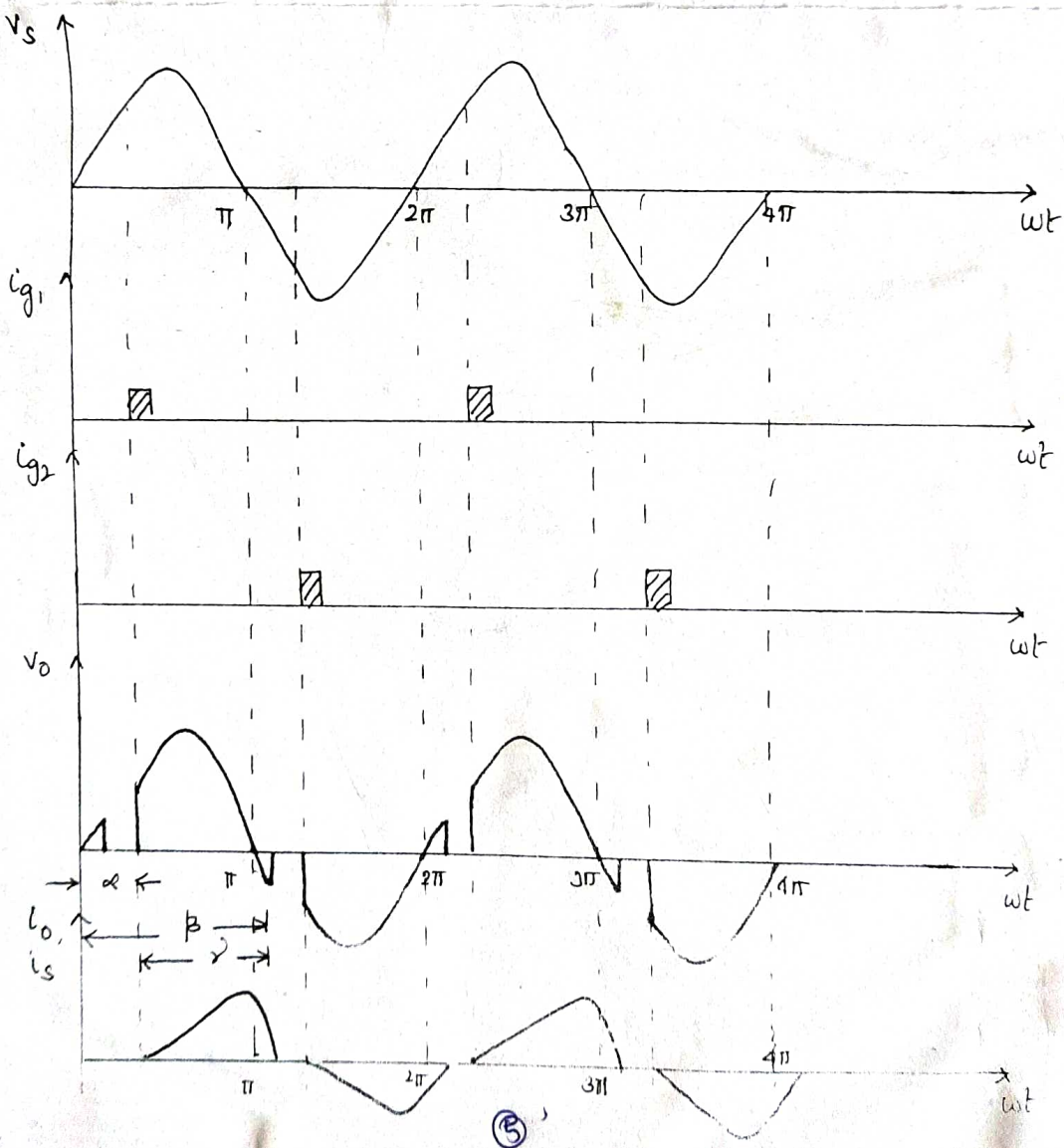
$$V_r = \left[\frac{1}{\pi} \int_{\alpha}^{\beta} V_m^2 \sin^2 \omega t \, d\omega t \right]^{1/2}$$

$$= \left[\frac{V_m^2}{\pi} \int_{\alpha}^{\beta} \frac{1 - \cos 2\omega t}{2} \, d\omega t \right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi} \left[\omega t - \frac{\sin 2\omega t}{2} \right]_{\alpha}^{\beta} \right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi} \left[\beta - \frac{\sin 2\beta}{2} - \alpha + \frac{\sin 2\alpha}{2} \right] \right]^{1/2}$$

$$V_r = \frac{V_m}{\sqrt{2\pi}} \left[\beta - \alpha + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2} \right]^{1/2}$$

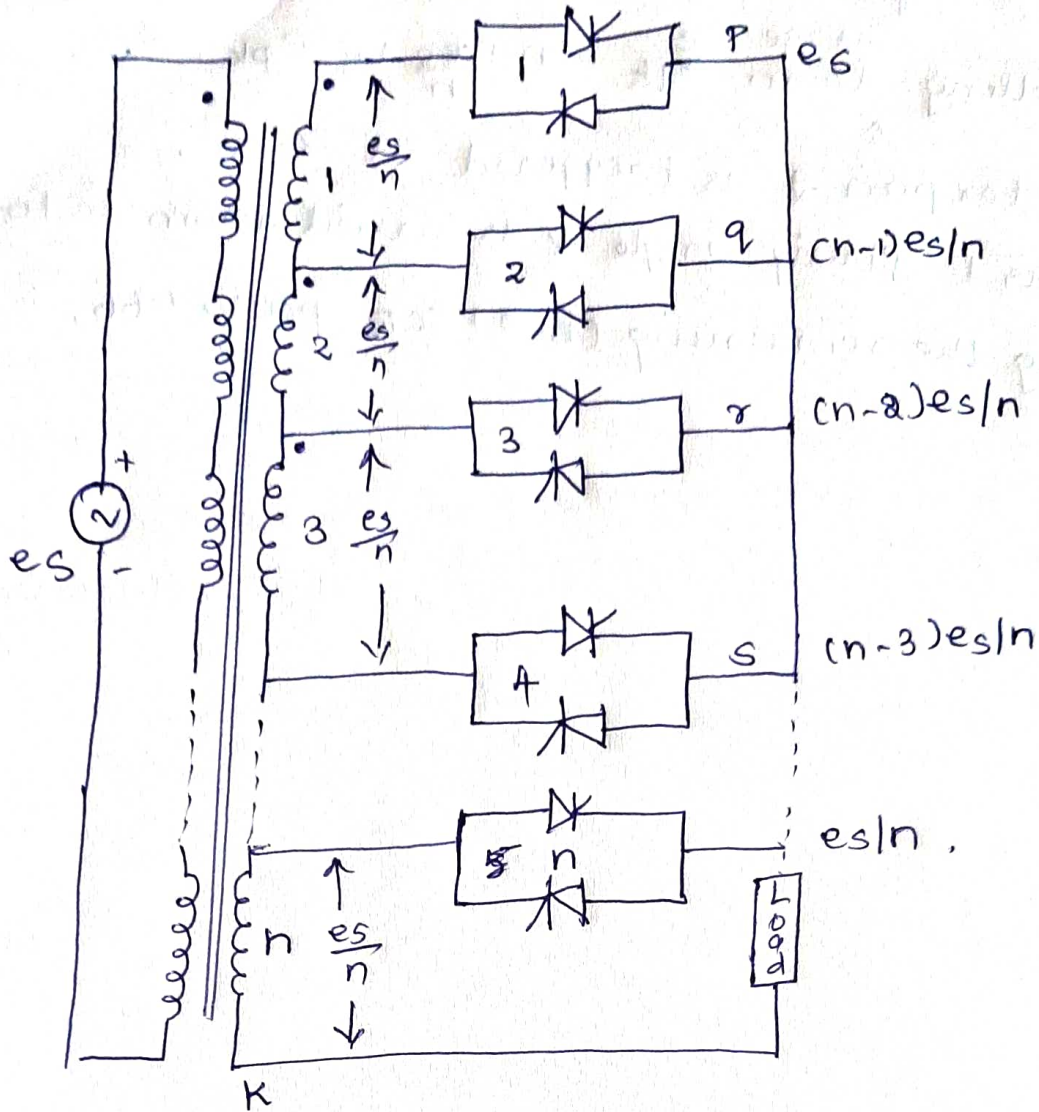


With RL load

Principle of operation :-

- \Rightarrow During 0 to π , S_1 is forward biased.
- \Rightarrow At $\omega t = \alpha$, S_1 is triggered and i_o starts building up through the load.
- \Rightarrow At $\omega t = \pi$, load & source voltage are zero, but the current is not zero due to the presence of inductance. So, load current extends upto β . angle β is called extinction Angle.
- after π , S_1 is reverse biased but does not turn off due to i_o is not zero.
- \Rightarrow when i_o is zero, S_1 is turned off.
- from β to $\pi + \alpha$, no current exists in the power circuit.
- \Rightarrow Thyristor S_2 turned on at $\omega t = \pi + \alpha$.
- at $\omega t = 2\pi$, V_s, V_o becomes zero, but i_{s2} is not zero due to inductive load.
- At $\omega t = \pi + \alpha + \beta$, $i_{s2} = 0$, S_2 is turned off.
- again at $2\pi + \alpha$, S_1 is turned on, and current start building up.

Multistage sequence Control of A.C Regulators



⇒ By using more than two stages of sequence control, it is possible to have further improvement in power factor and reduction in harmonics.

⇒ The transformer has n secondary windings, each secondary is rated for e_s/n , $e_s \rightarrow$ source voltage.

⇒ The voltage of node P with respect to K is e_s , voltage of terminal q is $(n-1)e_s/n$.

⇒ Voltage control from $e_{sk} = (n-3)\frac{e_s}{n}$ to $e_{sk} = (n-2)\frac{e_s}{n}$ is required,

thyristor pair A is triggered at $\omega t = 0^\circ$, firing angle of thyristor pair 3 is controlled from $\alpha = 0^\circ$ to 180° ,

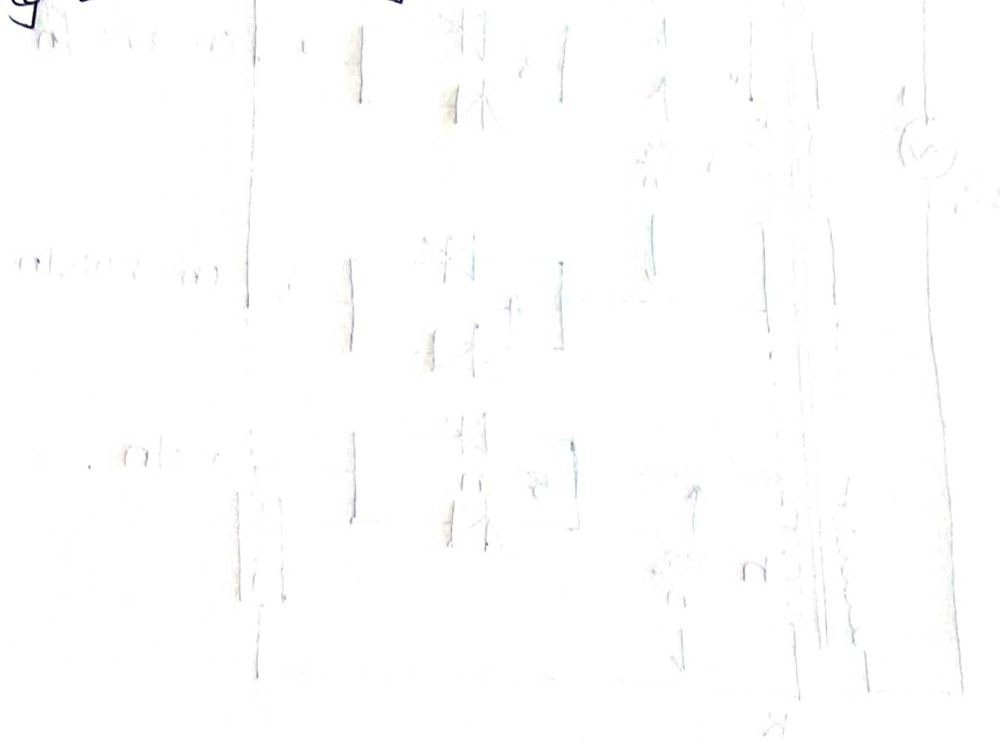
all other thyristor pairs are kept off.

=> controlling voltage from $e_{pk} = \frac{(n-1)e_s}{n}$ to $e_{pk} = e_s$,

thyristor pair 2 is triggered at $\alpha = 0^\circ$,

pair 1 firing angle α is varied from 0° to 180° ,

keeping the remaining $(n-2)$ SCR pairs off.



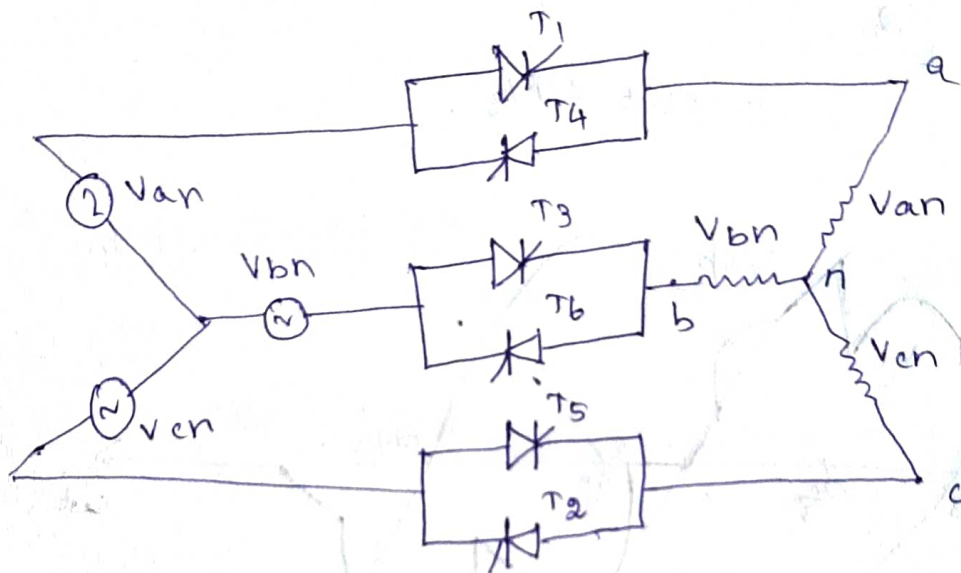
[Faint, mostly illegible handwritten notes, likely bleed-through from the reverse side of the page. Some words like 'thyristor', 'firing', and 'angle' are faintly visible.]

Three phase Bidirectional Delta Connected controllers:

Three Phase AC Regulators.

The 3 ϕ ac full wave controller with star connected load is shown in figure. If a neutral connection is made, load current can flow provided atleast one thyristor is conducting. At high power level, neutral connection is to be avoided. Because of load triplen currents, they may flow through the phase inputs and the neutral.

Without the neutral connected, each device would conduct for $\pi/2$ in the order T_1 to T_6 at $\pi/3$ apart.



\Rightarrow If the thyristor T_1 is triggered at α , then for a symmetrical 3 ϕ load voltage, the other trigger angles are T_3 at $\alpha + \frac{2\pi}{3}$ and T_5 at $\alpha + \frac{4\pi}{3}$. For the antiparallel devices, T_4 is at $\alpha + \pi$, T_6 at $\alpha + \frac{5\pi}{3}$, T_2 at $\alpha + \frac{7\pi}{3}$.

(i) $0 \leq \omega t \leq \pi/3$:-

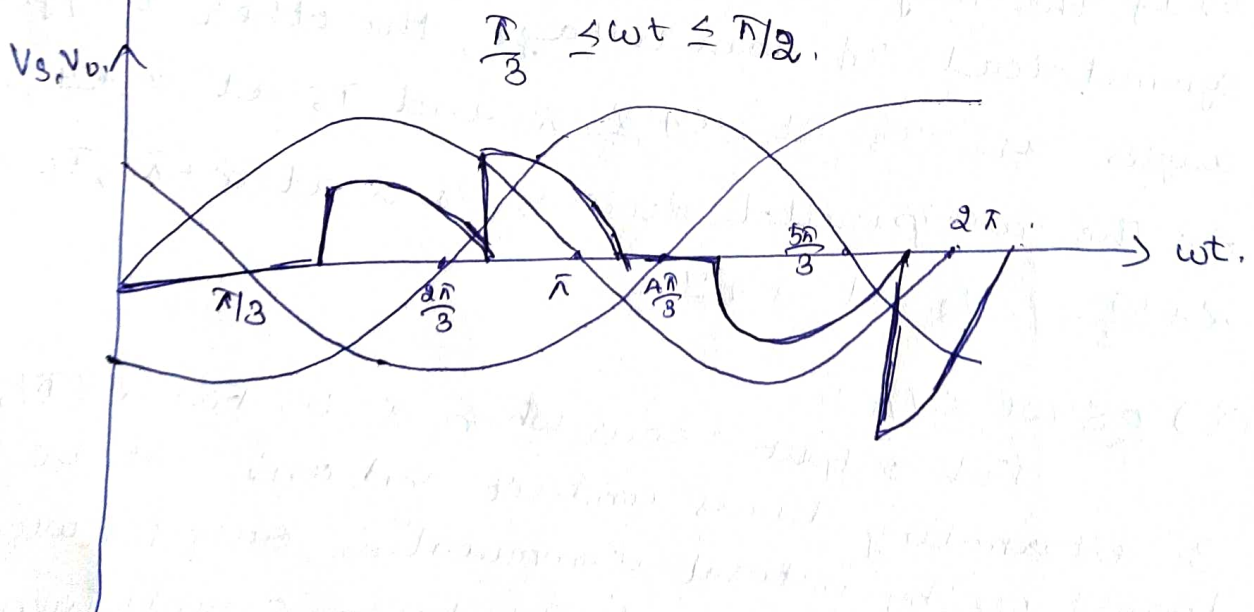
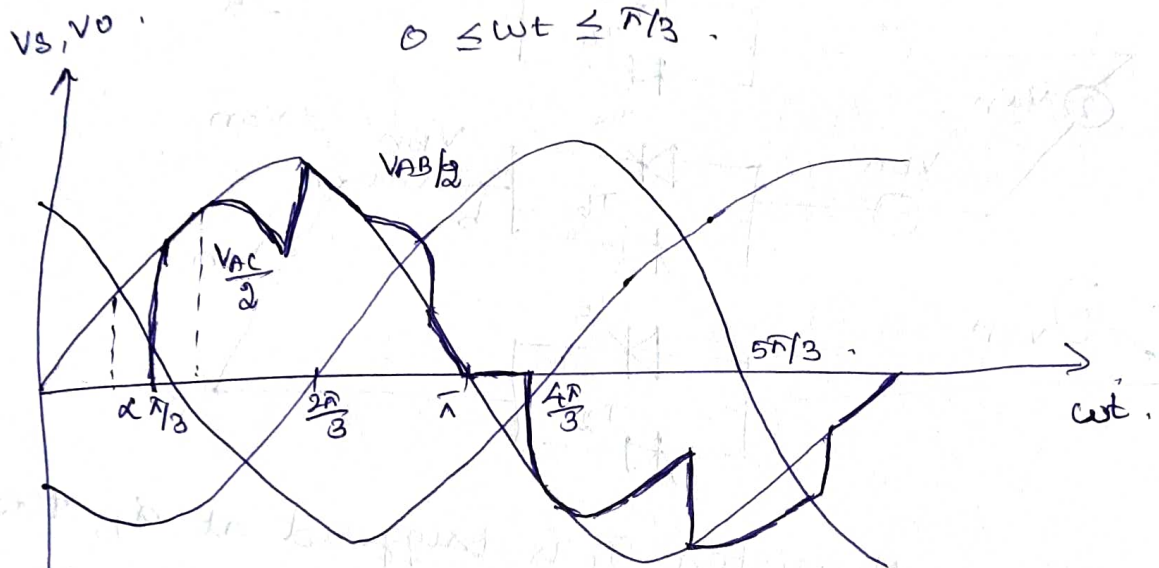
Full output occurs when $\alpha = 0$. For $\alpha \leq \pi/3$, 3 alternating devices conduct and one will be turned off by natural commutation, only for $\omega t \leq \pi/3$, can three sequential devices be on simultaneously.

(ii) $\pi/3 \leq \omega t \leq \pi/2$

The turning on of one device naturally commutate another conducting device and only two phase can be conducting, that is only two thyristors conduct at any time. Line to neutral voltage waveform for $\alpha = \pi/3$ & $\alpha = \pi/2$ shown in fig.

3 thyristor conduct, the voltage is of the form $\frac{V_{mL}}{\sqrt{3}} \sin \omega t$.

when 2 thyristor conduct, the voltage is of the form $\frac{V_{mL}}{2} \sin(\phi - \pi/2)$.



For $\alpha = \pi/4$, the rms load voltage/phase is,

$$V_{\text{rms}} = V_{\text{ml}} \left[\frac{1}{\pi} \left(\int_{\alpha}^{\pi/3} \frac{\sin^2 \omega t}{(\sqrt{3})^2} d\omega t + \int_{\pi/3}^{\pi/3+\alpha} \frac{\sin^2 (\omega t - \pi/6)}{2 \cdot 2} d\omega t \right. \right. \\ \left. \left. + \int_{\pi/3+\alpha}^{2\pi/3} \frac{\sin^2 \omega t}{(\sqrt{3})^2} d\omega t + \int_{2\pi/3}^{2\pi/3+\alpha} \frac{\sin^2 (\omega t - \pi/6)}{2 \cdot 2} d\omega t + \int_{2\pi/3+\alpha}^{\pi} \frac{\sin^2 \omega t}{(\sqrt{3})^2} d\omega t \right)^{1/2} \right. \\ \left. + \int_{\frac{2\pi}{3}}^{\frac{2\pi}{3}+\alpha} \frac{\sin^2 (\omega t - \pi/6)}{2 \cdot 2} d\omega t \right]^{1/2}$$

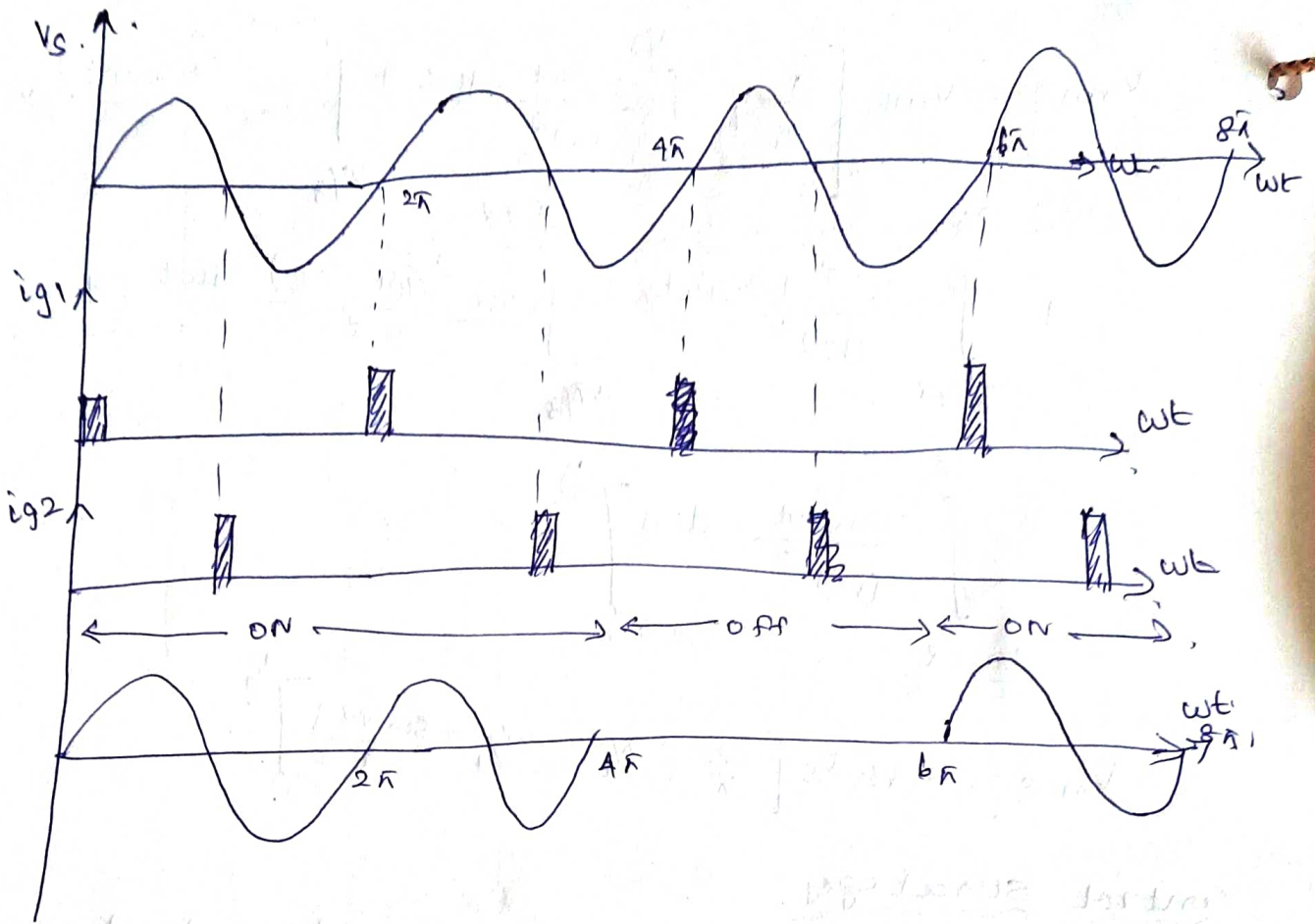
$$V_{\text{rms}} = \sqrt{6} V_s \left[\frac{1}{\pi} \left(\pi/6 - \alpha/4 + \frac{\sin 2\alpha}{8} \right) \right]^{1/2}$$

Control Strategy :-

Power factor control : Integral cycle control.

In industry for some applications, almost no variation in speed of the control is achieved by connecting load to source for some cycles, and then disconnecting the load for some off cycles. It is called integral cycle control.

The source energizes the load for n cycles, when gate pulses are withdrawn, load remains off for m cycles. By varying the numbers of n & m cycles, power delivered to load can be regulated.



RMS load voltage

$$V_{rms} = \frac{1}{\text{Periodicity}} \int_0^{2\pi} V_m^2 \sin^2 \omega t \, d\omega t \text{ (1st cycle)} + \int_0^{2\pi} V_m^2 \sin^2 \omega t \, d\omega t \text{ (2nd cycle)} + \dots + \int_0^{2\pi} V_m^2 \sin^2 \omega t \, d\omega t \text{ (nth cycle)}$$

For \$n\$ on cycle, \$m\$ off cycle, periodicity \$= (n+m)2\pi\$,

$$V_r = \frac{1}{2\pi(n+m)} \cdot n \left[\int_0^{2\pi} V_m^2 \sin^2 \omega t \, d\omega t \right]^{1/2}$$

$$= \left[\frac{n V_m^2}{4\pi(n+m)} \int_0^{2\pi} (1 - \cos 2\omega t) \, d\omega t \right]^{1/2}$$

$$= \left[\frac{n \cdot V_m^2}{4\pi(n+m)} [2\pi] \right]^{1/2}$$

$$V_r = \frac{V_m}{\sqrt{2}} \sqrt{\frac{n}{n+m}}$$

Take $k = \sqrt{\frac{n}{n+m}}$.

$V_r = V_s \sqrt{k}$.

RMS load current

$I_r = \frac{V_r}{R}$

$= \frac{V_s \sqrt{k}}{R}$

Power delivered to the load :

$P = \frac{V_r^2}{R} = \frac{V_s^2 k}{R}$

Power factor $\frac{V_r^2 / R}{V_s \times I_r} = \frac{V_r^2 / R}{V_s \times \frac{V_r}{R}}$

$= \frac{V_s \sqrt{k}}{V_s}$

$P.F = \sqrt{k}$

Multi stage sequence control :-

Two stage sequence control :-

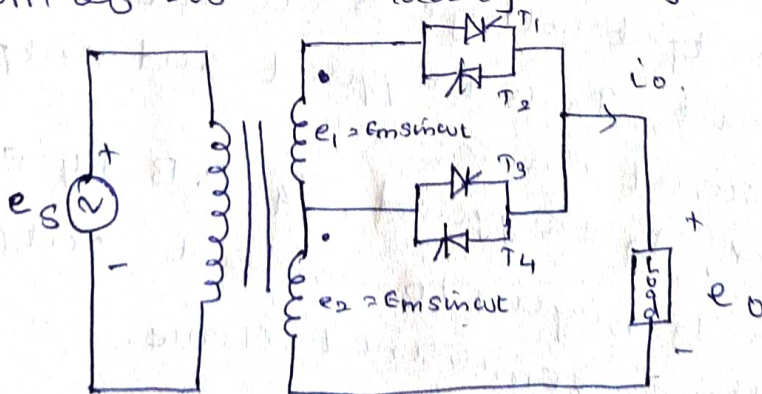
⇒ Two stage sequence control of a.c regulators

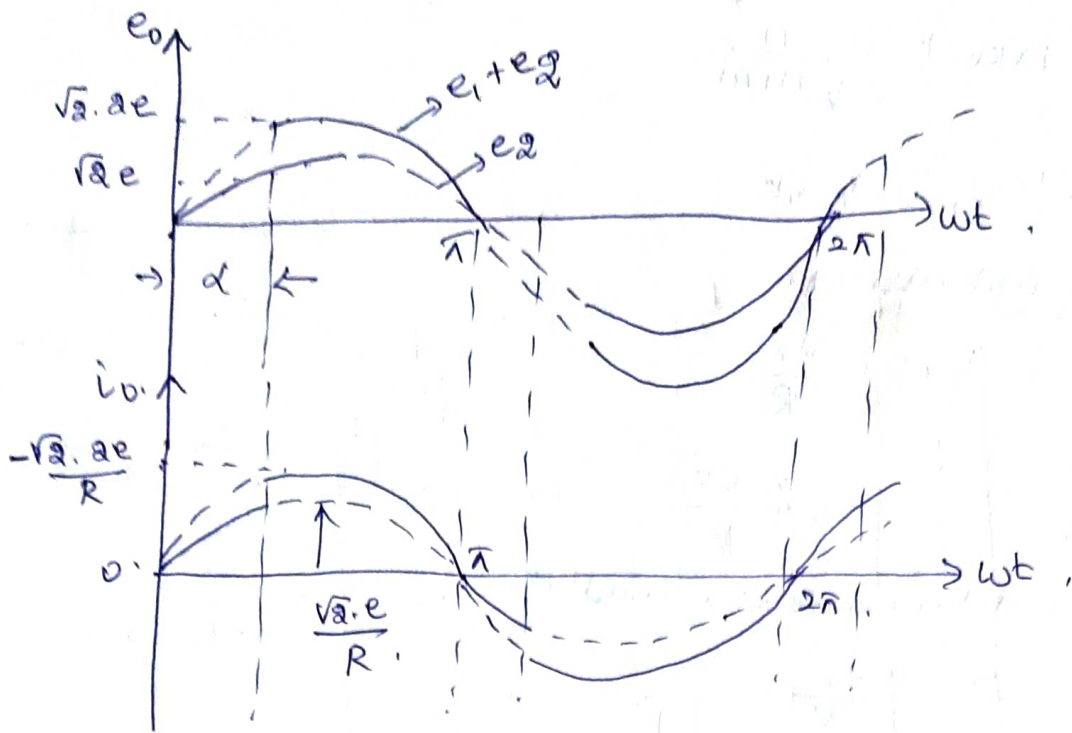
employs two stages in parallel.

⇒ For source voltage $e_s = E_m \sin \omega t$,

$e_1 = e_2 = E_m \sin \omega t$.

sum of two secondary voltages is $2 E_m \sin \omega t$.





\Rightarrow For R load, load current waveform is identical with output voltage waveform.

\Rightarrow When both pairs T_1, T_2 & T_3, T_4 are in operation, firing angle for T_3, T_4 is zero,

T_1, T_2 varied from 180° to 0 .

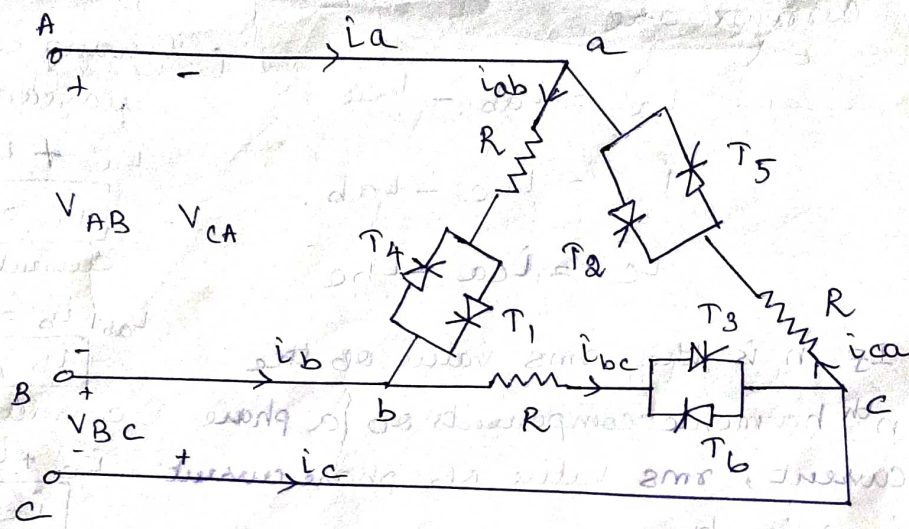
to obtain output voltage E to $2E$.

$\Rightarrow T_3$ triggered at $wt = 0$, follows $e_a = E_m \sin wt$ curve. when SCR T_1 is triggered at $wt = \alpha$, voltage e_1 reverse biases T_3 , it is turned off, T_1 begins to conduct, output voltage jumps from e_a to $(e_1 + e_a)$ and follows $2E_m \sin wt$ curve.

\Rightarrow at $wt = \pi$, output voltage & current are zero. at this instant, T_4 is triggered and output voltage follows $E_m \sin wt$ curve.

\Rightarrow at $wt = \pi + \alpha$, T_2 triggered, T_4 R.B by $E_m \sin \alpha$, so, T_4 off. T_2 begins to conduct, output voltage follows $2E_m \sin wt$ curve.

Three phase Bidirectional Delta connected controllers:



load may be connected in delta, The phase current in a normal 3 ϕ is only $\frac{1}{\sqrt{3}}$ of the line current.

Instantaneous line-to-line voltages are

$$V_{AB} = \sqrt{2} V_s \cdot \sin \omega t$$

$$V_{BC} = \sqrt{2} V_s \cdot \sin \left(\omega t - \frac{2\pi}{3} \right)$$

$$V_{CA} = \sqrt{2} V_s \cdot \sin \left(\omega t - \frac{4\pi}{3} \right)$$

For resistive loads, rms output phase voltage can be determined from,

$$V_o = \left[\frac{1}{\pi} \int_{\alpha}^{\pi} V_{ab}^2 d(\omega t) \right]^{1/2}$$

$$= \left[\frac{1}{\pi} \int_{\alpha}^{\pi} (\sqrt{2} V_s \sin \omega t)^2 d(\omega t) \right]^{1/2}$$

$$= \left[\frac{1}{\pi} \int_{\alpha}^{\pi} 2 V_s^2 \sin^2 \omega t \cdot d(\omega t) \right]^{1/2}$$

$$= \frac{\sqrt{2} V_s}{\sqrt{\pi}} \left[\int_{\alpha}^{\pi} \frac{1 - \cos 2\omega t}{2} d(\omega t) \right]^{1/2}$$

$$V_{orms} = \frac{V_s}{\sqrt{\pi}} \left[\omega t - \frac{\sin 2\omega t}{2} \right]_{\alpha}^{\pi} = \frac{V_s}{\sqrt{\pi}} \left[\pi - \alpha + \frac{\sin 2\alpha}{2} \right]^{1/2}$$

line currents can be determined from the phase currents are,

$$i_a = i_{ab} - i_{ca}$$

$$i_b = i_{bc} - i_{ab}$$

$$i_c = i_{ca} - i_{bc}$$

(∴ using KCL in circuit diagram, current

$$i_a + i_{ca} = i_{ab}$$

$$i_a = i_{ab} - i_{ca}$$

current at node b,

$$i_{ab} + i_b = i_{bc}$$

$$i_b = i_{bc} - i_{ab}$$

current at node c,

$$i_c + i_{bc} = i_{ca}$$

$$i_c = i_{ca} - i_{bc}$$

If n is the rms value of the n^{th} harmonic components of a phase current, rms value of phase current is given by,

$$I_{ab} = \left[I_1^2 + I_3^2 + I_5^2 + I_7^2 + \dots + I_n^2 \right]^{1/2}$$

Due to delta connection, the triplen harmonic components of the phase currents would blow around the delta and would not appear in the line. Hence,

$$I_a = \sqrt{3} \left[I_1^2 + I_5^2 + I_7^2 + \dots + I_n^2 \right]^{1/2}$$

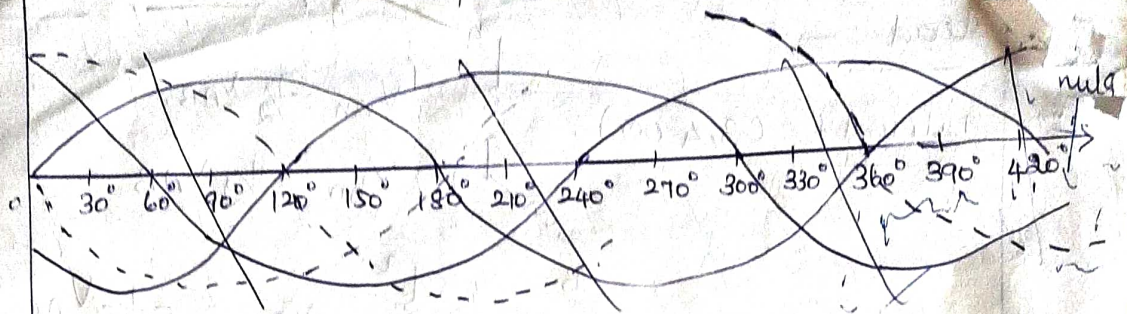
$$I_a < \sqrt{3} I_{ab}$$

⇒ SCR are rated to carry phase currents and withstand the line voltage.

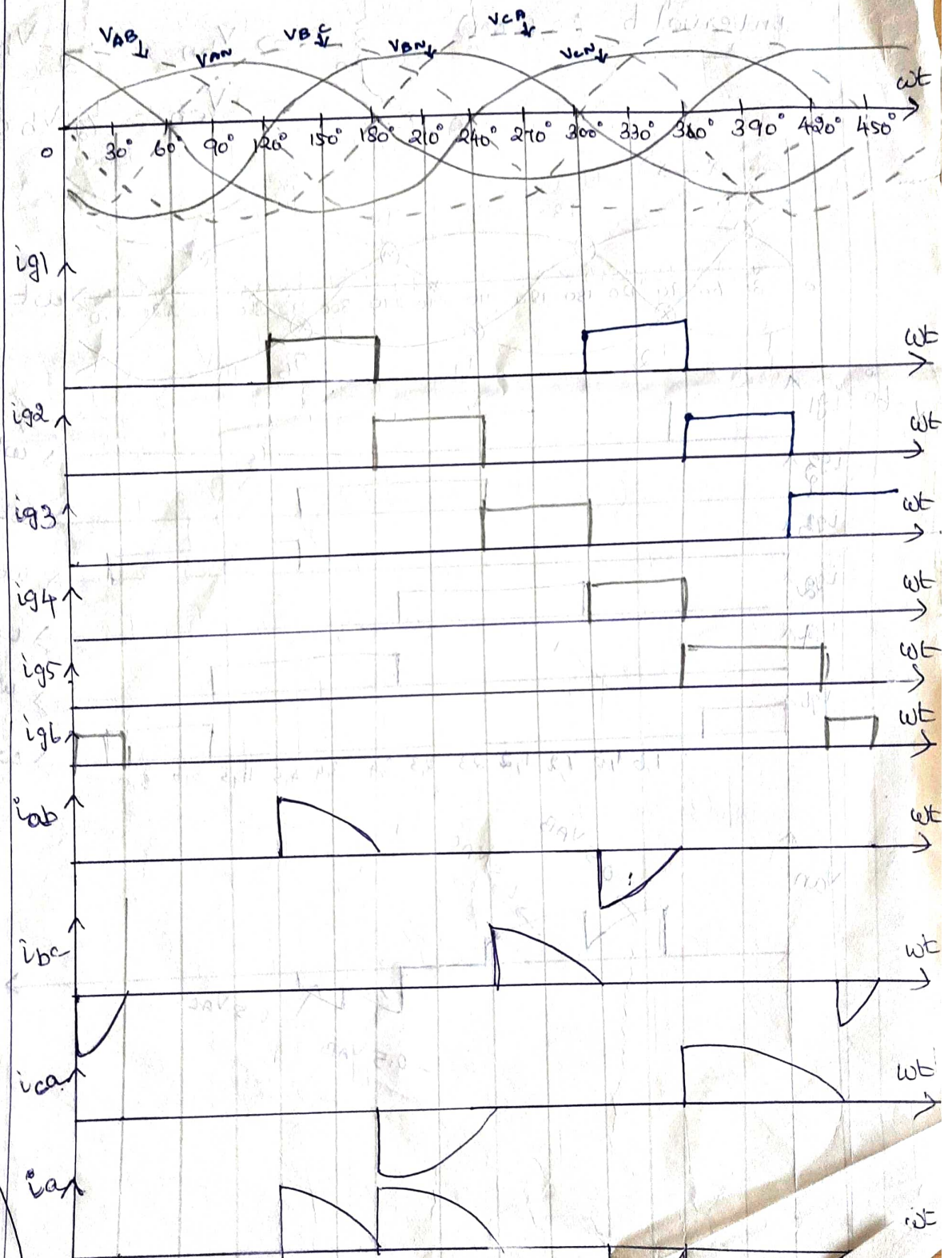
⇒ The power factor is slightly higher.

⇒ The voltage across an R-load is the corresponding line-to-line voltage when one SCR in that phase is on.

⇒ The firing angle α is measured from the zero crossing of the line-to-line voltage and the SCR turned on in the sequence they are numbered.



3 ϕ Bidirectional delta connected controllers:-



$\alpha > 180^\circ$

Draw
 i_b, i_c
 ab, bc

A device which converts input power at one frequency to output power at a different frequency is called a cycloconverter.

Two types :-

- (i) Step down cycloconverters [output frequency f_o is lower than supply frequency f_s].
 (ii) Step up cycloconverters.
 [$f_o > f_s$].

Applications :-

- (i) speed control of high power ac drives
 (ii) Induction heating
 (iii) static VAR compensation.
 (iv) Power supply in aircraft or shipboards.

Cycloconverter is to provide either a variable frequency power from a fixed input frequency power (ac motor speed control) or a fixed frequency power from a variable input frequency power (aircraft or shipboard power supplies or wind generators).

single phase to single phase
Principle of step up cycloconverter operation :-

Types :-

- (i) mid point cycloconverter
 (ii) Bridge type cycloconverter.

load is assumed to be resistive.

Step up cycloconverter requires forced commutation.

(1) Mid point cycloconverter :-

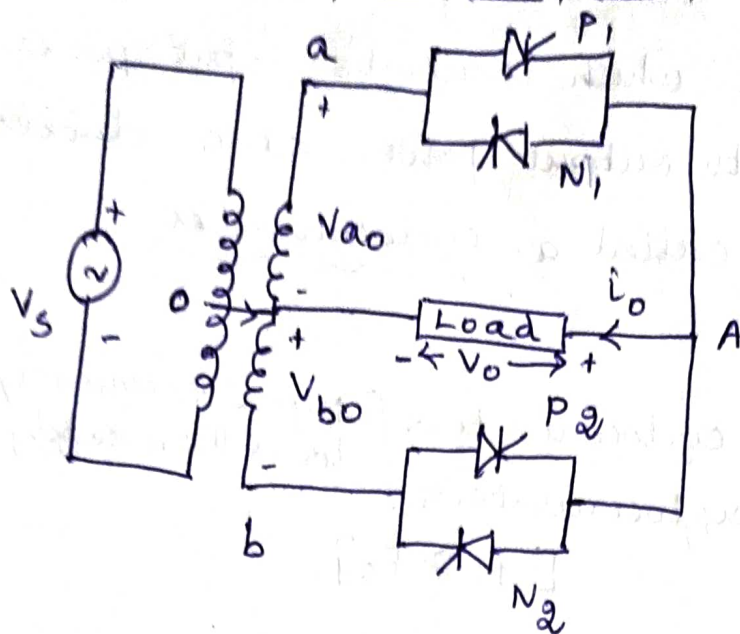


fig (1).

- ⇒ It consists of a 1ϕ transformer with mid tap on the secondary winding and 4 thyristors.
- ⇒ Two of these thyristors P_1, P_2 are for +ve group and the other two N_1, N_2 are for -ve group.
- ⇒ Load is connected between secondary winding mid point o and terminal A.

operation :-

1) During the +ve half cycle of supply voltage, both SCRS P_1 & N_2 are F.B, i.e terminal a is +ve, b is -ve. From $\omega t = 0^\circ$ to $\omega t = \pi$.

P_1 turned ON at $\omega t = 0^\circ$, $V_o = +ve$ with $A \rightarrow +ve$, $o \rightarrow -ve$. load voltage follows the +ve envelop of the supply voltage.

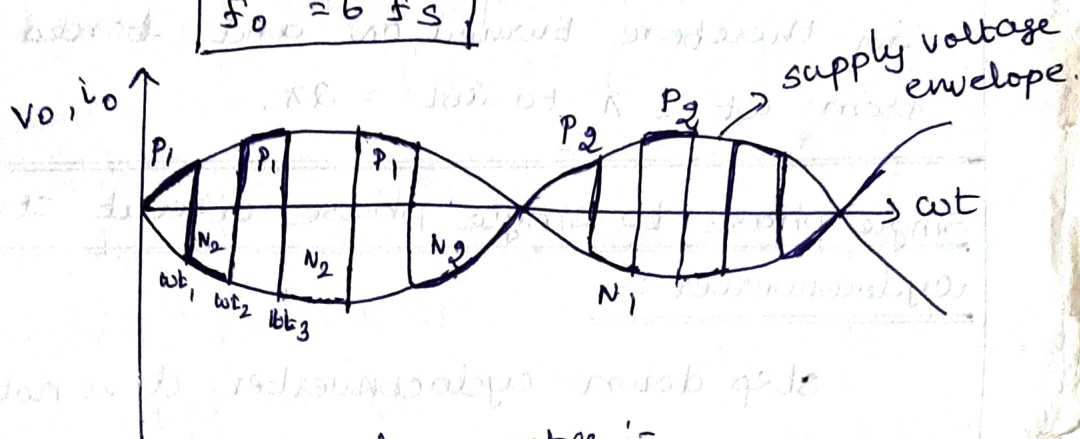
P_1 forced commutated at $\omega t = \omega t_1$, N_2 turned ON, $V_o = -ve$, $A \rightarrow -ve$, $o \rightarrow +ve$. load voltage follows the -ve envelop of the supply voltage.

N_2 forced commutated at $\omega t = \omega t_2$, $P_1 \rightarrow ON$.

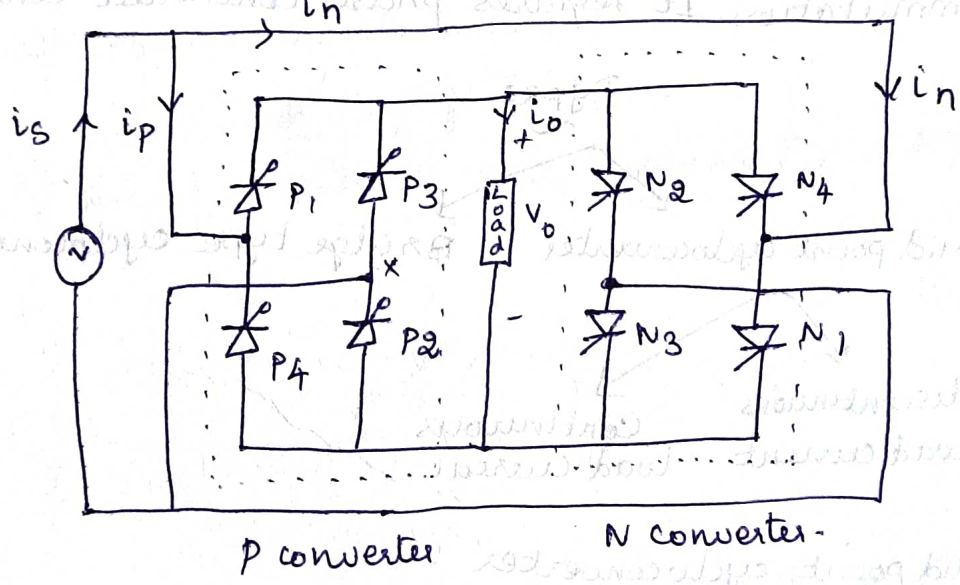
d) During the -ve half cycle, $\omega t = \pi$,
 b is +ve, a is -ve. Both SCRs P_2 & N_1 are F.B.

P_2 turned on at $\omega t = \pi$,
 at $\omega t = \frac{1}{2f_s} + \frac{1}{2f_s}$; $P_2 \rightarrow$ forced commutate
 $N_1 \rightarrow$ turned on.

$$f_0 = 6 f_s$$



cii) Bridge type cycloconverter :-



It consists of 8 thyristors,
 P_1 to P_4 for +ve group, N_1 to N_4 for -ve group.
 During +ve half cycle of supply voltage,
 P_1, P_2, N_1, N_2 are F.B, from $\omega t = 0^\circ$ to $\omega t = \pi$.
 P_1, P_2 are turned on at $\omega t = 0^\circ$, load voltage \rightarrow +ve,
 $A \rightarrow$ +ve, $O \rightarrow$ -ve. V_0 follows the +ve envelop
 of supply voltage.

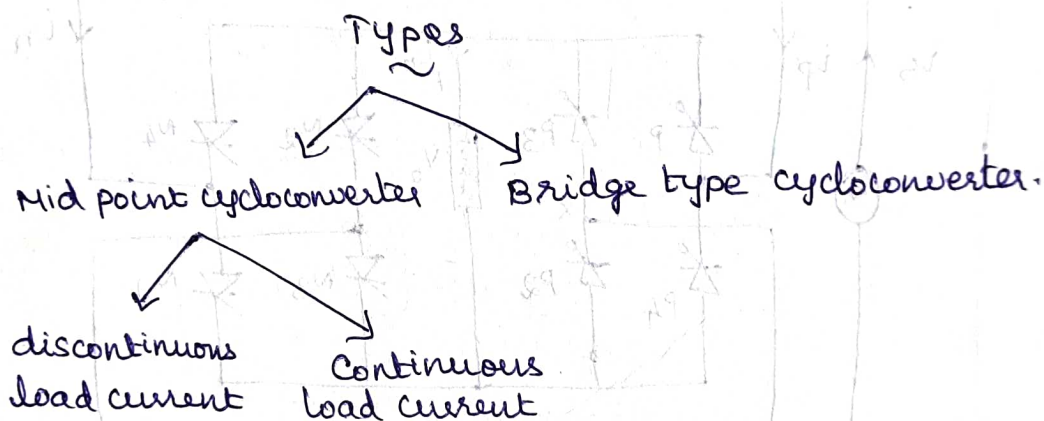
⇒ at ωt_1 , pair P_1, P_2 is force commutated, N_1, N_2 is turned ON, $V_o \rightarrow -ve$, $0 \rightarrow +ve$, $A \rightarrow -ve$. Load voltage follows the negative envelop of source voltage.

⇒ at ωt_2 , $N_1, N_2 \rightarrow F.C$, $P_1, P_2 \rightarrow ON$.

after $\omega t = \pi$, P_3, P_4, N_3, N_4 are F.B. These can therefore turned ON and forced commutated from $\omega t = \pi$ to $\omega t = 2\pi$.

Single phase to single phase circuit step down cycloconverter :-

step down cycloconverter does not require forced commutation, need only line or natural commutation. It requires phase controlled converters.



Mid point cycloconverter :-

The load is assumed to be RL

[Circuit diagram shown in fig (1)].

Discontinuous load current :-

SCR P_1 triggered at $\omega t = \alpha$, load current i_o starts building up in the positive direction from A to O.

P_1 naturally commutated at $\omega t = \beta$,

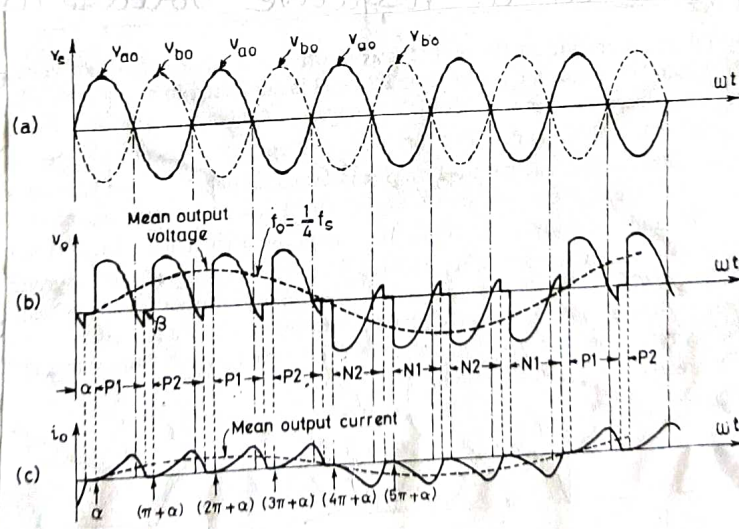
Load current i_o becomes zero, at $\omega t = \beta > \pi$, but less than $\pi + \alpha$.

During negative half cycle, b is +ve, o is -ve. P_2 triggered at $\omega t = \pi + \alpha$, at $\omega t = \pi + \beta$, i_o decays to zero, $P_2 \rightarrow$ Naturally commutated. $\omega t = 2\pi + \alpha$, P_1 again turned on, load current is seen to be discontinuous.

After 4 positive half cycles of load voltage and load current, N_2 is gated at $4\pi + \alpha$, $o \rightarrow +ve$, $b \rightarrow -ve$. Load current direction reversed, from o to A . N_1 is gated at $5\pi + \alpha$, $N_2 \rightarrow$ off, load voltage, load current is obtained for 4 negative half cycles.

$$f_o = \frac{1}{4} f_s$$

P_1 goes to blocking state before P_2 is gated.



Step down cycloconverter with discontinuous load current.

Continuous load current :-

when $\alpha \rightarrow +ve$, $0 \rightarrow -ve$. P_1 triggered
at $\omega t = \alpha$, positive output voltage appears across
load and load current.

at $\omega t = \pi$, supply and load voltages are zero,

$P_1 \rightarrow R.B$, load current is continuous,

P_1 Not turned off at $\omega t = \pi$,

P_2 triggered at $\omega t = \pi + \alpha$, reverse voltage
appear across ~~P_1~~ P_1 , and it is turned off.

when P_1 is commutated, load current has built
up to a value equal to $R.R$, P_2 turned on
load current builds up further than $R.R$.

$\omega t = (2\pi + \alpha)$, $P_1 \rightarrow ON$, $P_2 \rightarrow OFF$.

current through P_1 builds up beyond $R.S$.

\Rightarrow ~~at~~ At the end of 4 +ve half cycles of o/p
voltage, load current is $R.U$.

\Rightarrow when N_2 is now triggered after P_2 , load
is subjected to a negative voltage cycle and
load current i_o decreases from $R.U$ to AB ^(negative).

$\Rightarrow N_1$ is gated at $5\pi + \alpha$, i_o becomes more
negative than AB at $(6\pi + \alpha)$.

\Rightarrow supply voltage gone through four cycles.

$$f_o = \frac{1}{4} f_s$$

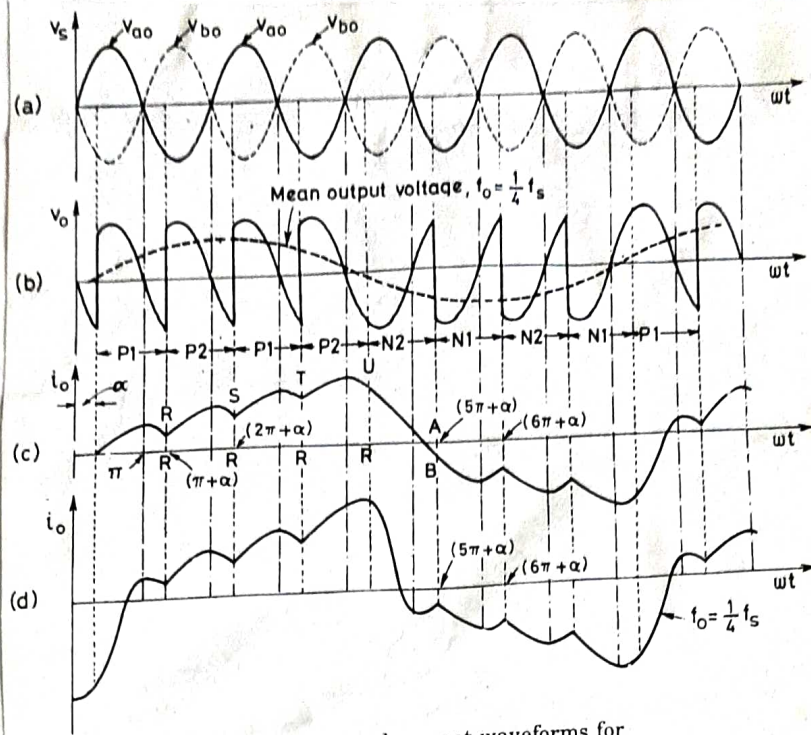


Fig. 10.4. Voltage and current waveforms for step-down cycloconverter with continuous load current.

Three-phase to Single-phase Cyclo-converter

(NPTEL notes)

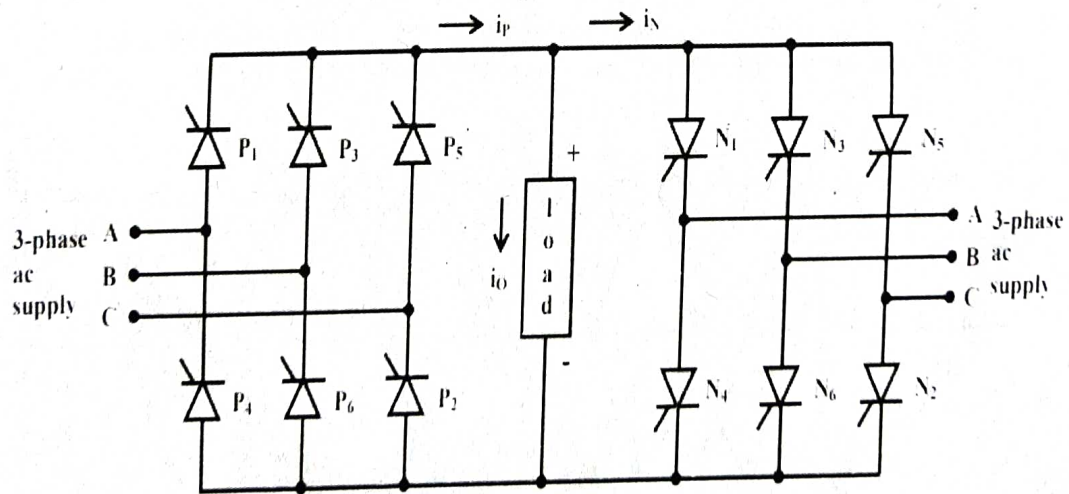


Fig. 30.1: Three-phase to single-phase cycloconverter

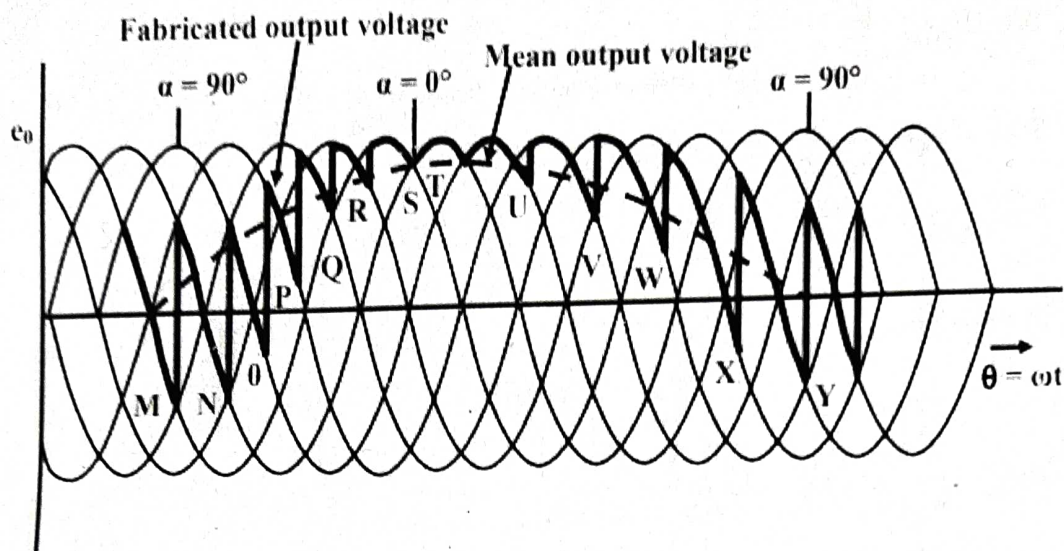


Fig. 30.2 Output voltage waveforms for a three-phase to single phase cyclo-converter.

Non-circulating current mode of operation

1) Two three-phase full-wave (six-pulse) bridge converters (rectifier) connected back to back, with six thyristors for each bridge, are used. The ripple frequency here is 300 Hz, six times the input frequency of 50 Hz.

2) One converter – bridge 1 (positive) or bridge 2 (negative), conducts at a time, but both converters do not conduct at the same time.

3) The sequence of conduction of the thyristors is 1 & 6, 4 & ⁵2, 3 & 2, and so on. When thyristor 1 is triggered, the conducting thyristor (#5) in top half, being reverse biased at that time, turns off. Similarly, when thyristor 2 is triggered, the conducting thyristor (#6) in bottom half, being reverse biased at that time turns off. This sequence is repeated in cyclic order. So, natural or line commutation takes place in this case.

4) The firing angle (α) of two converters is first decreased starting from the initial value of 90 degree to the final value of 0°, and then again increased to the final value of 90°, as shown in Fig. 30.2. Also, for positive half cycle of the output voltage waveform, bridge 1 is used while bridge 2 is used for negative half cycle.

5) The initial value of firing angle delay is kept at $\alpha_1 = 90^\circ$, such the average value (dc) of the output voltage in this interval of near $V_{av} \cos 90$ is zero.

The next thyristor in sequence is triggered at $\alpha_2 < 90^\circ$, as the firing angle is decreased for each segment, to obtain higher voltage. This can be observed from the points, M, N, O, P, Q, R & S,

6) The first quarter cycle of the output voltage waveform from 0° to 90°, is

obtained. The second quarter cycle of the above waveform from 90° to 180° , is obtained using the segments starting from the point T, U, V, W, X & Y

7) To obtain the negative half cycle of the output voltage waveform ($180^\circ - 360^\circ$), the other bridge converter (#2) termed negative (N) is used in the same manner as given earlier, i.e. its firing angle delay (α) is first decreased starting from the initial value 90° to the final value of 0° , and then again increased 90° . The two half cycles (positive and negative) together give one complete cycle ($0^\circ - 360^\circ$) of the output voltage waveform.

Circulating Current Mode of Operation

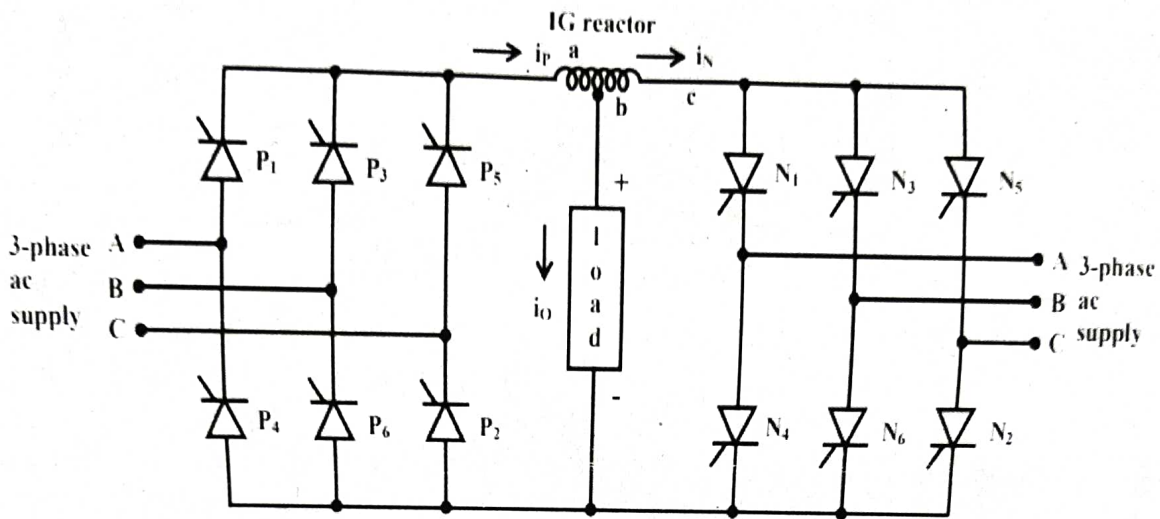


Fig. 30.4: Cycloconverter (circulating current mode) with Inter-group (IG) reactor

Circulating current mode of operation of the cyclo-converter, both the converters would conduct at a time, with an inter-group reactor (IGR) between the positive and negative groups as shown in Fig. 30.4. It may be noted that, though the output voltages of two converters in the same phase have the same average value, but their output voltage waveforms as a function of time are, however, different, and as a result, there is a net potential difference (voltage) across two converters. Due to this voltage, the reactor is inserted to limit the circulating current.

Bimbhra

Three-phase to Three-phase Cyclo-converter

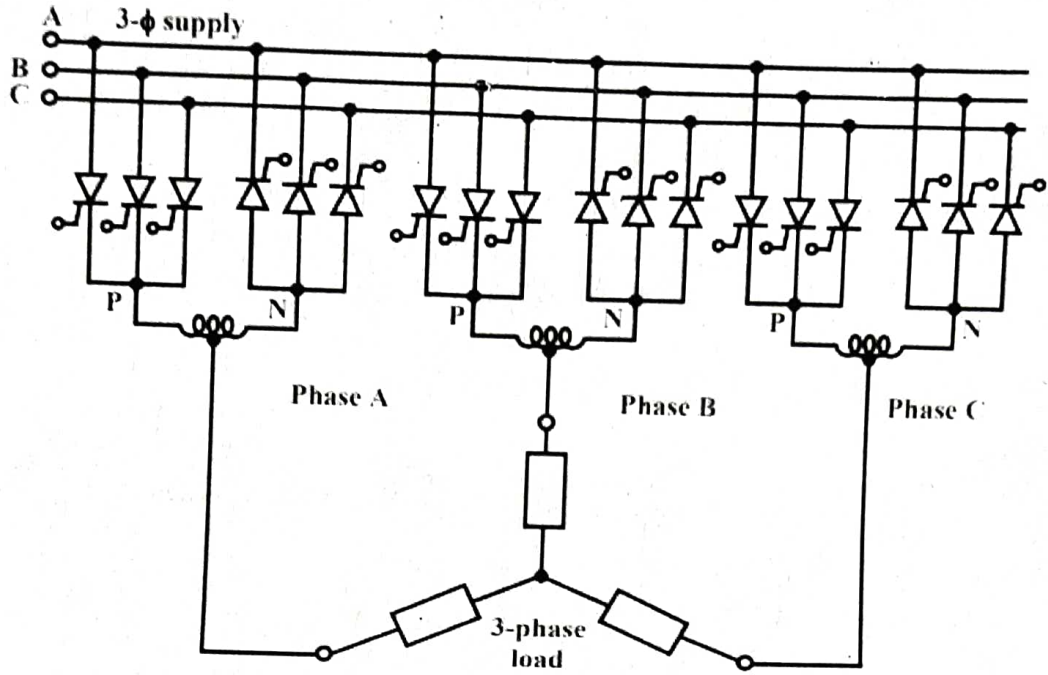
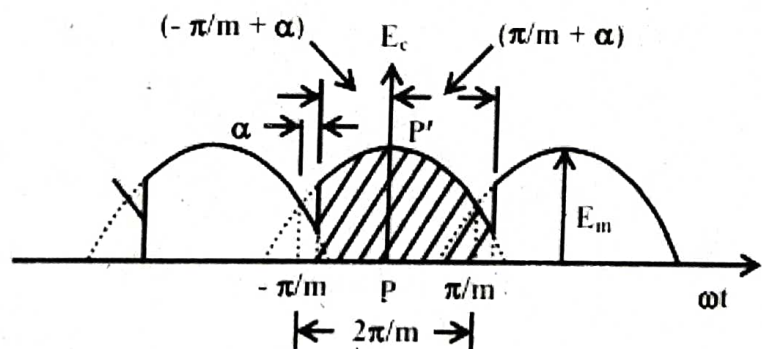


Fig. 31.1: Three-phase to three-phase cycloconverter

- 1) The total number of thyristors used is 18, thus reducing the cost of power components, and control circuits needed to generate the firing pulses for the thyristors.
- 2) The circulating current mode of operation is used, in which both (positive and negative) converters in each phase, conduct at the same time. Inter-group reactor in each phase as shown, is needed here.
- 3) The firing sequence of the thyristors for the phase groups, B & C are same as that for phase group A, but lag by the angle, 120° and 240° , respectively. Thus, a balanced three-phase voltage is obtained at the output terminals, to be fed to the three-phase load.
- 4) The average value of the output voltage is changed by varying the firing angles (α) of the thyristors, whereas its frequency is varied by changing the time interval ($T/3 = 1/(3f_0)$), after which the next (incoming) thyristor is triggered.

Analysis of the Cyclo-converter Output Waveform



2: Output voltage waveform for m-phase converter with firing angle α

In a 3ϕ half wave converter, each phase conducts for $\frac{2\pi}{3}$ radians.

for m phase half wave converter, each phase conducts for $\frac{2\pi}{m}$ radians.

(Bimbha)

Instantaneous phase voltage is,

$$V = V_{mp} \cos \omega t = \sqrt{2} V_{ph} \cos \omega t.$$

for $\alpha = 0^\circ$, conduction takes place from $-\frac{\pi}{m}$ to $\frac{\pi}{m}$.

for any firing angle α , conduction is from

$$\left(-\frac{\pi}{m} + \alpha\right) \text{ to } \left(\frac{\pi}{m} + \alpha\right).$$

$$\begin{aligned} V_{dc} &= \frac{m}{2\pi} \int_{-\pi/m + \alpha}^{\pi/m + \alpha} V_{mp} \cos \omega t \cdot d(\omega t) \\ &= V_{mp} \times \frac{m}{2\pi} \left[-\sin \omega t \right]_{-\pi/m + \alpha}^{\pi/m + \alpha} \end{aligned}$$

$$= V_{mp} \left[\frac{m}{\pi} \sin \frac{\pi}{m} \right] \cos \alpha$$

for zero firing angle delay, the average value of direct voltage V_{do} is given by,

$$V_{do} = \sqrt{2} V_{ph} \left(\frac{m}{\pi} \right) \sin \frac{\pi}{m}$$

when $\alpha = 90^\circ$, $V_{do} = 0$.

Fundamental rms value per phase is given by,

$$V_{or} = \frac{V_{do}}{\sqrt{2}} = \frac{V_{mp}}{\sqrt{2}} \times \frac{m}{\pi} \cdot \sin \frac{\pi}{m}$$

Matrix Converter :-

⇒ Matrix converter uses nine bi-directional switches so arranged that any of the three input phases can be connected to any output phase.

⇒ Voltage at any input terminal may be made to appear at any output terminal while the current in any phase of the load may be drawn from any phase or phases of the input supply.

⇒ For the switches, power MOSFET, IGBTs or transistor have been used.

⇒ The switches controlled by, at any time one and only one of the 3 switches connected to an output phase must be closed to prevent "short circuiting".

⇒ only 27 switch combinations are allowed

⇒ physical limits is maximum peak to peak output voltage cannot be greater than the minimum voltage difference between two phases of the input.

The voltages V_{an} , V_{bn} , V_{cn} at the output terminals are related to the input voltages V_{Ao} , V_{Bo} , V_{Co} as,

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} S_{Aa} & S_{Ba} & S_{Ca} \\ S_{Ab} & S_{Bb} & S_{Cb} \\ S_{Ac} & S_{Bc} & S_{Cc} \end{bmatrix} \begin{bmatrix} V_{Ao} \\ V_{Bo} \\ V_{Co} \end{bmatrix}$$

$S_{Aa} - S_{Cc}$ are switching variables of the corresponding switches.

For a balanced linear star-connected load at the output terminals, input phase currents are related to the output phase currents by,

$$\begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} S_{Aa} & S_{Ab} & S_{Ac} \\ S_{Ba} & S_{Bb} & S_{Bc} \\ S_{Ca} & S_{Cb} & S_{Cc} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

⇒ Maximum voltage transfer ratio is 0.866.

3 types of methods

i) Venturini method

ii) PWM

iii) Space vector modulation.

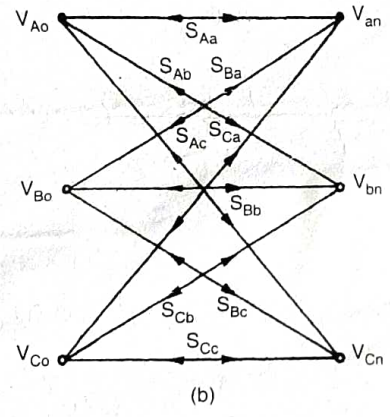
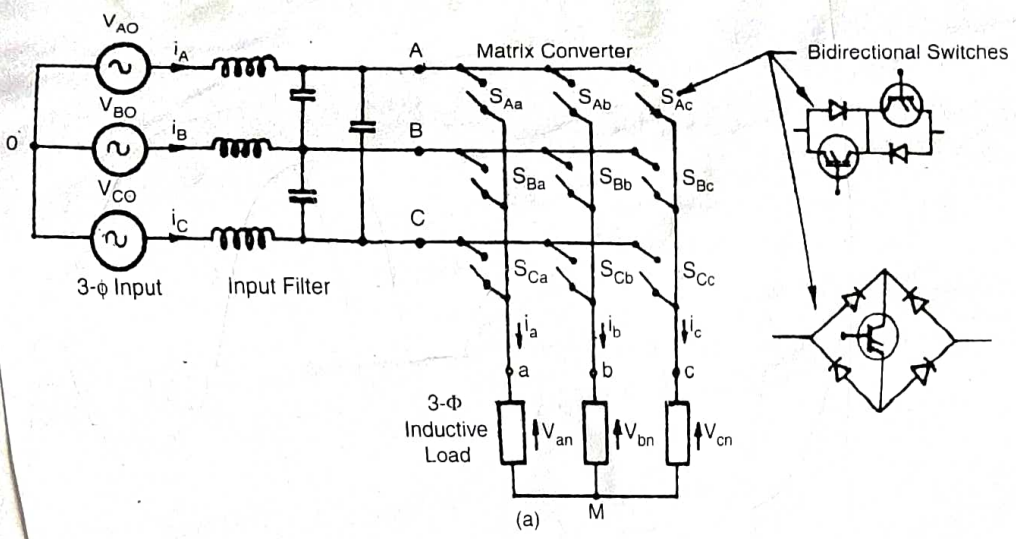
Advantages :-

- (1) Inherent Bidirectional power flow
- (2) sinusoidal input-output waveforms with moderate switching frequency.
- (3) Compact design.

limitations :-

- (1) Now availability of the fully controlled switches
- (2) Complex control
- (3) Commutation and protection of the switches

A. K. Chatterjee



(a) 3φ-3φ Matrix converter (forced commutated cycloconverter) circuit with input filter and (b) switching